Chapter 9 Transient Response

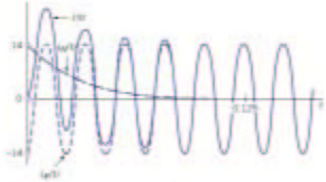
Chapter 9: Outline

Transient Response (in Time)

The response before steady state.

Affected by Natural response and Forced Response

(by **either** or **both** energy sources)



$$y(t) = y_N(t) + y_F(t) = \sum_{i=1}^{n} A_i e^{a_i t} + y_F(t)$$

↓

First Order Circuit

Zero input, step, Pulse, Switched DC, Switched AC

↓

Second Order Circuit

Zero input → Over-damped (two real distinct roots),

Under damped (complex conjugate roots),

Critically damped (repeated roots)

↓

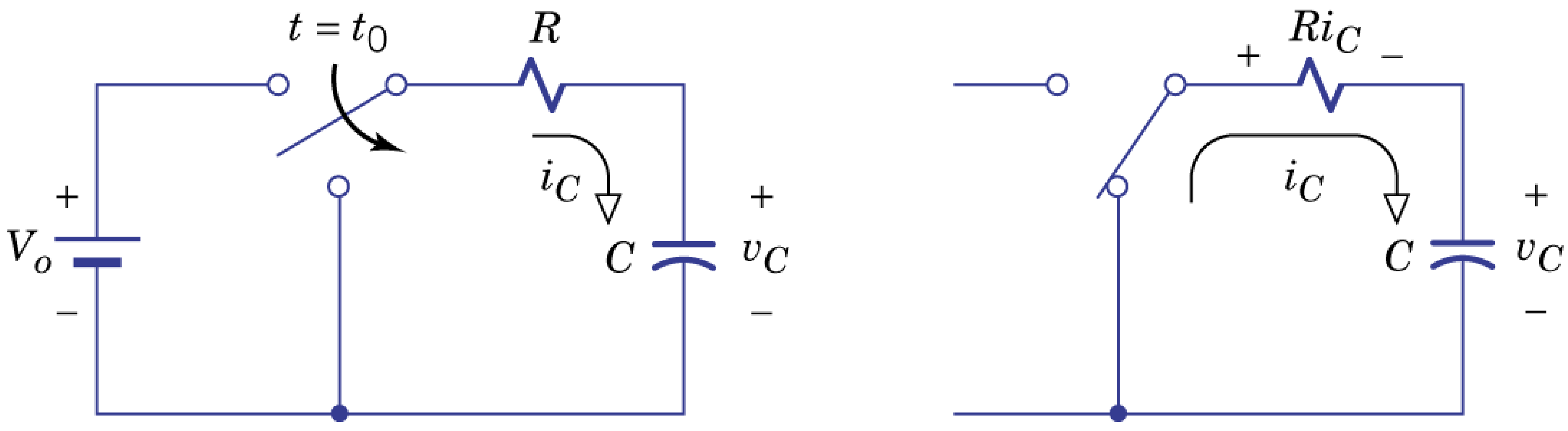
Switched DC

First-Order Transients

First-Order Circuits

- First order circuits: Circuits contain only one inductor or one capacitor, governed by first-order differential equations.
- Zero-input response: the circuit has no applied source after a certain time. It is determined by natural response and the initial condition.
- Zero-state response: the circuit has no initial stored energy.

RC First-Order Circuits



(a) Switched *RC* circuit

(b) Circuit for $t > t_0$

$$\text{At } t = t_0, v_C = V_0, w_C = \frac{1}{2} C V_0^2, i_C = 0$$

$$\text{For } t > t_0, \text{zero input response } RCv_C' + v_C = 0$$

$$\text{characteristic equation: } RCs + 1 = 0$$

$$s = -\frac{1}{RC} \equiv -\frac{1}{t} \quad (t \text{ : time constant})$$

RC First-Order Circuits

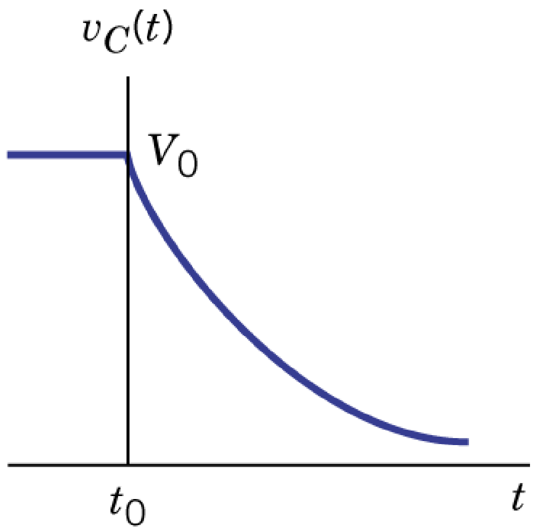
$$v_C(t) = Ae^{-t/t}, \quad t > t_0$$

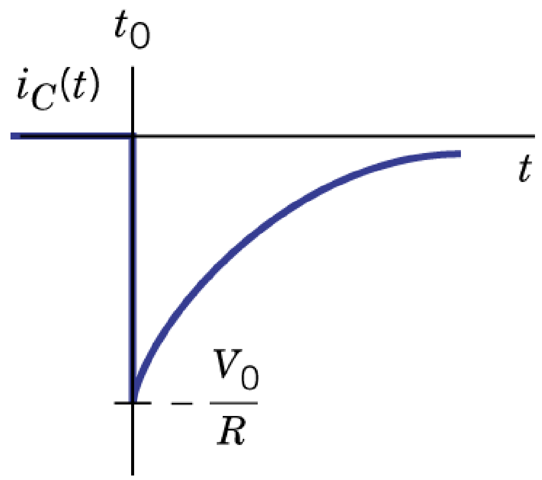
$$v_C(t_0^+) = v_C(t_0^-) = V_0 = Ae^{-t_0/t}$$

$$A = V_0 e^{t_0/t}$$

$$v_C(t) = V_0 e^{-(t-t_0)/t}, t > t_0$$

$$i_C(t) = Cv_C' = -\frac{V_0}{R} e^{-(t-t_0)/t}, t > t_0$$





RC First-Order Circuits

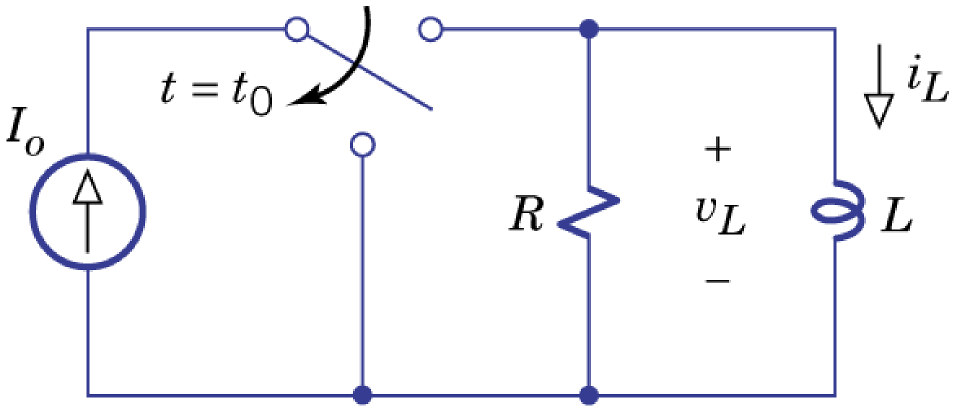
Dissipated power by the resistor *R*

$$p_R = Ri_C^2 = \left(\frac{V_0^2}{R}\right)e^{-2(t-t_0)/RC}, t > t_0$$

$$w_R = \int_{t_0}^{\infty} p_R dt = \frac{1}{2}CV_0^2$$

Capacitor's initial stored energy

RL First-Order Circuits
(structural dual of the RC circuit)



$$\left(\frac{L}{R}\right) i_L' + i_L = 0$$

characteristic equation : $\left(\frac{L}{R}\right) s + 1 = 0$

$$s = -\frac{R}{L} \equiv -\frac{1}{t} \quad (t = \frac{L}{R} : \text{time constant})$$

$$i_L(t) = I_0 e^{-(t-t_0)/t}, t > t_0$$

$$i_L(t_0^+) = i_L(t_0^-) = I_0$$

$$v_L(t) = L i_L' = -R I_0 e^{-(t-t_0)/t}, t > t_0$$

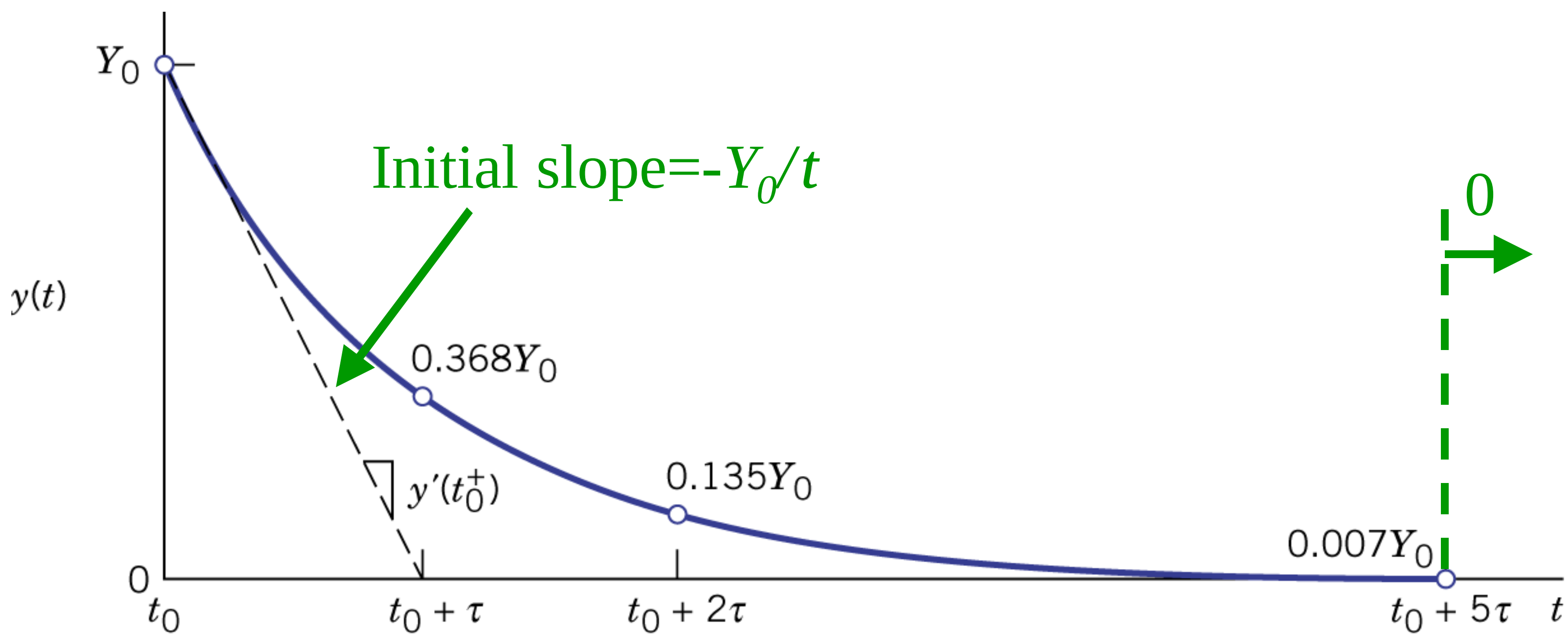
General First-Order Circuits

$$t = \begin{cases} R_{eq}C \\ L/R_{eq} \end{cases}$$

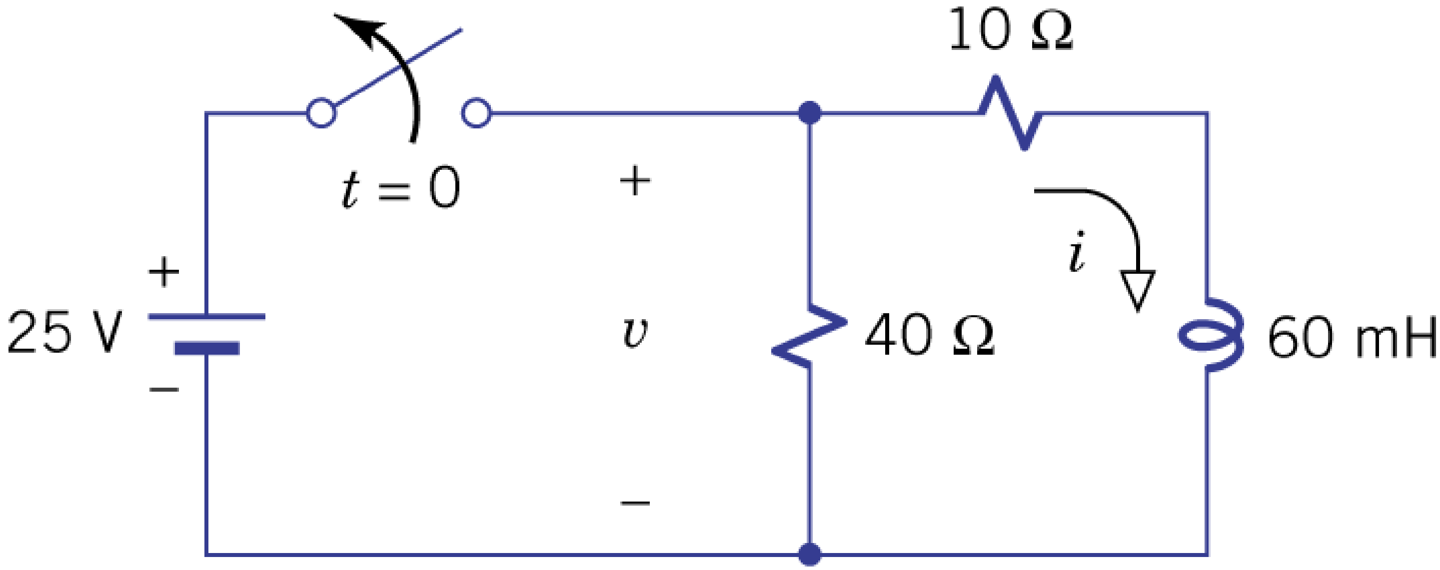
(If τ<0, then the circuit is unstable.)

zero input response : $y(t) = Y_0 e^{-(t-t_0)/t}, t > t_0$

$Y_0 = y(t_0^+)$, (capacitor voltage or inductor current)



Example 9.1: Zero-Input Response



$$R_{eq} = 10 + 40(\Omega)$$

$$t = L / R_{eq} = 1.2ms$$

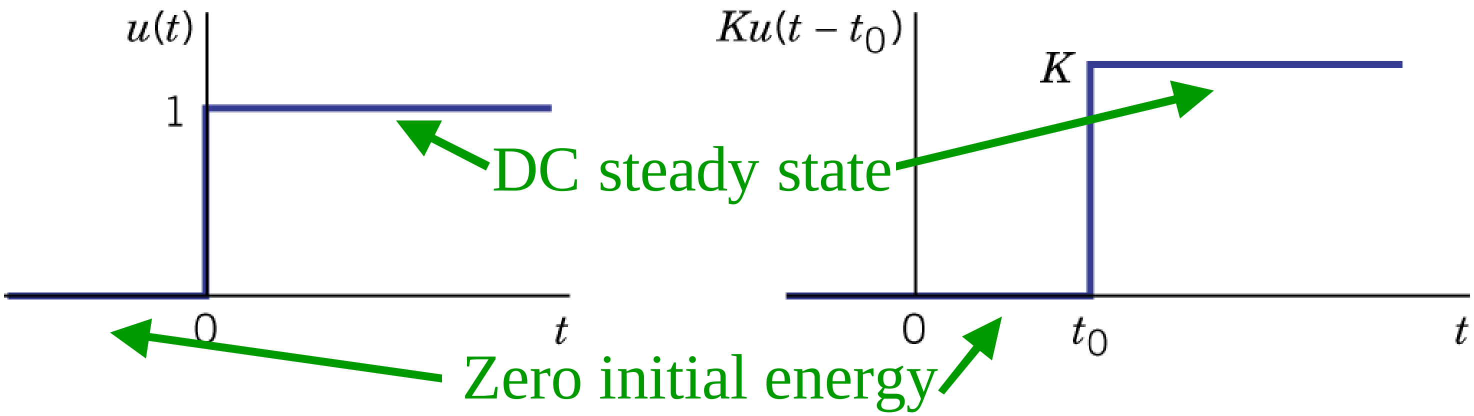
$$i(0^{+}) = i(0^{-}) = 2.5A$$

$$i(t) = 2.5e^{-t/t} A, t > 0$$

$$v(t) = -40i(t) = -100e^{-t/t} V, t > 0$$

Step Response

- Step response: response to a step input (OFF to ON) with zero initial conditions.

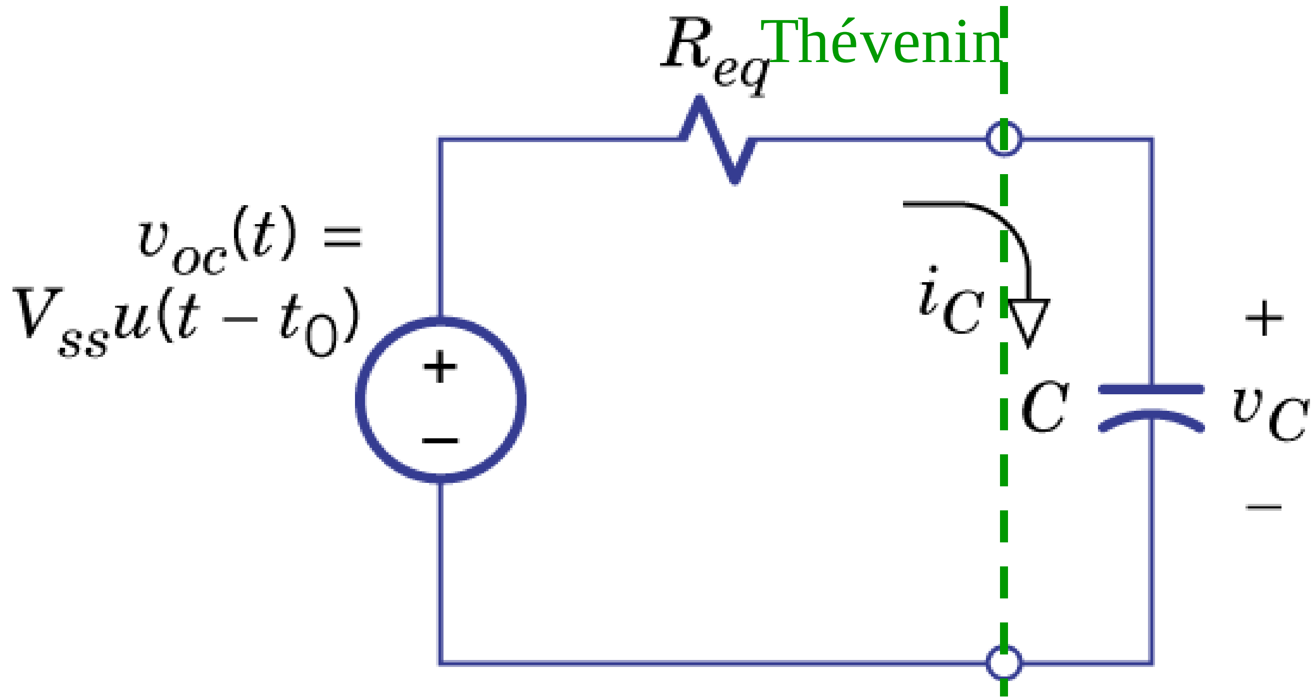


(a) The unit step $u(t)$ (b) $Ku(t-t_0)$

$$u(t) = \begin{cases} 0, t < 0 \\ 1, t > 0 \end{cases} \qquad Ku(t-t_0) = \begin{cases} 0, t - t_0 < 0 \\ K, t - t_0 > 0 \end{cases}$$

For a linear time-invariant circuit (LTI), the response to $Ku(t-t_0)$ is simply $Ky(t-t_0)$, where $y(t)$ is the unit step response.

First-Order RC Circuit



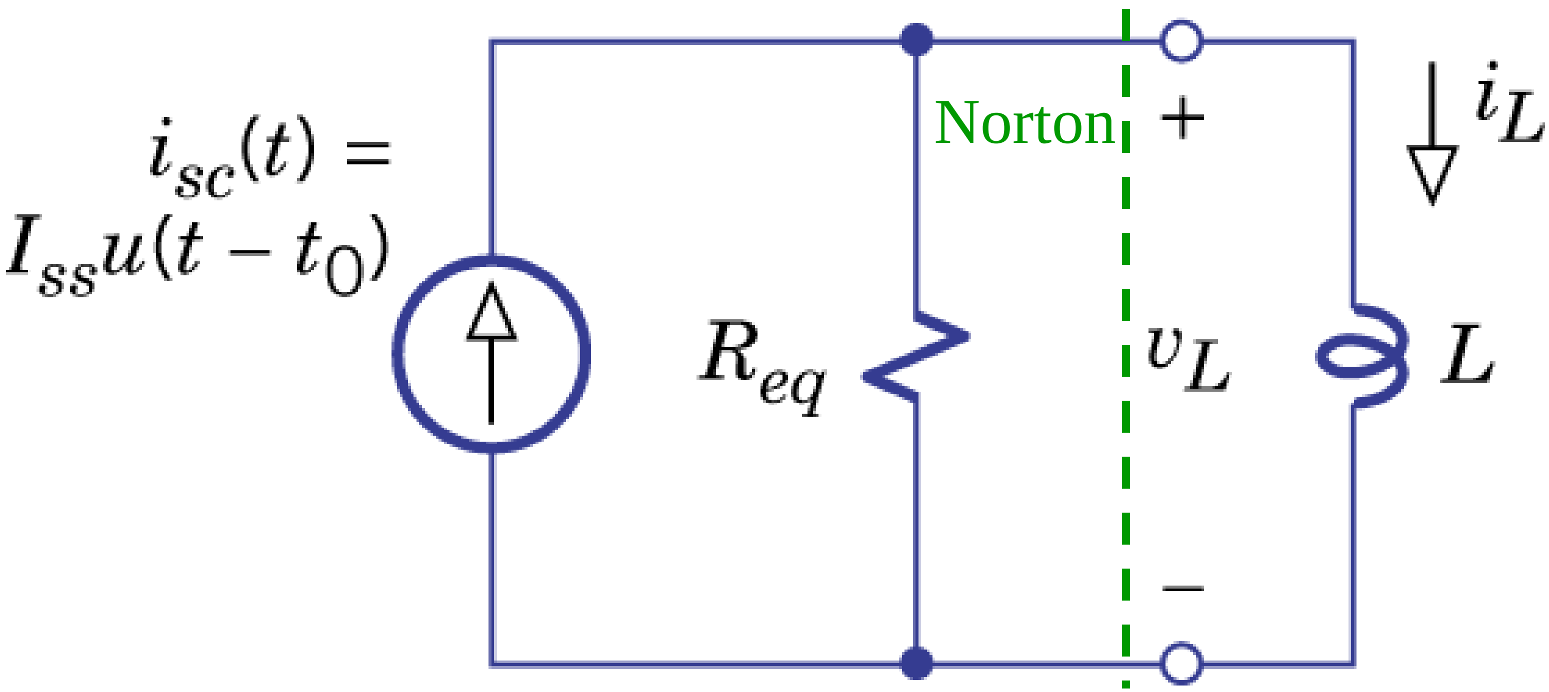
$$t v_C' + v_c = V_{SS} \text{ (nonhomogeneous diff. eq.)}$$

$$v_C(t) = V_{SS} + Ae^{-t/t}, \quad t = R_{eq}C$$

initial condition $v_C(t_0^+) = 0 \Rightarrow A = -V_{SS}e^{t_0/t}$

$$v_C(t) = V_{SS}\left[1 - e^{-(t-t_0)/t}\right], t > t_0, \qquad V_{SS} : \text{steady state voltage}$$

First-Order RL Circuit



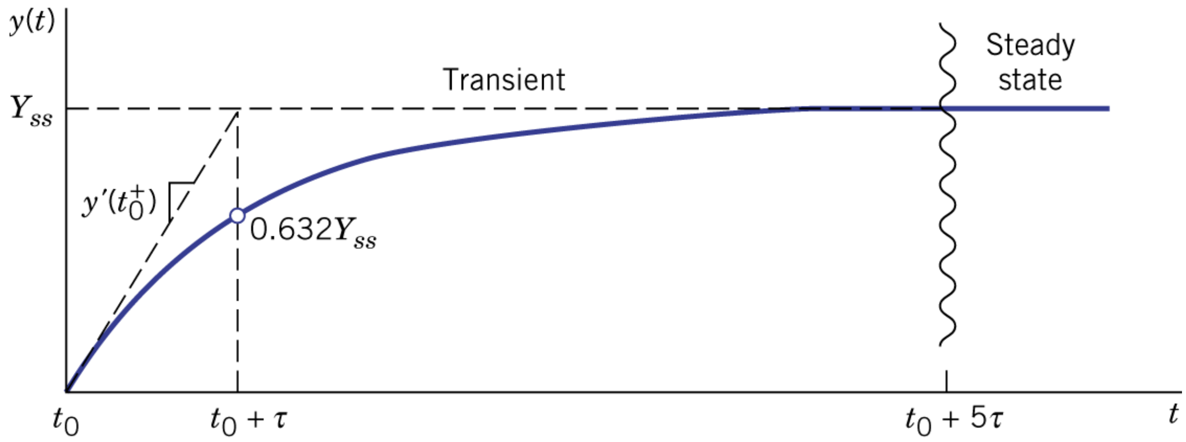
$$i_{SC}(t) = I_{SS}u(t - t_0)$$

$$i_L(t) = I_{SS}\left[1 - e^{-(t-t_0)/t}\right], t > t_0$$

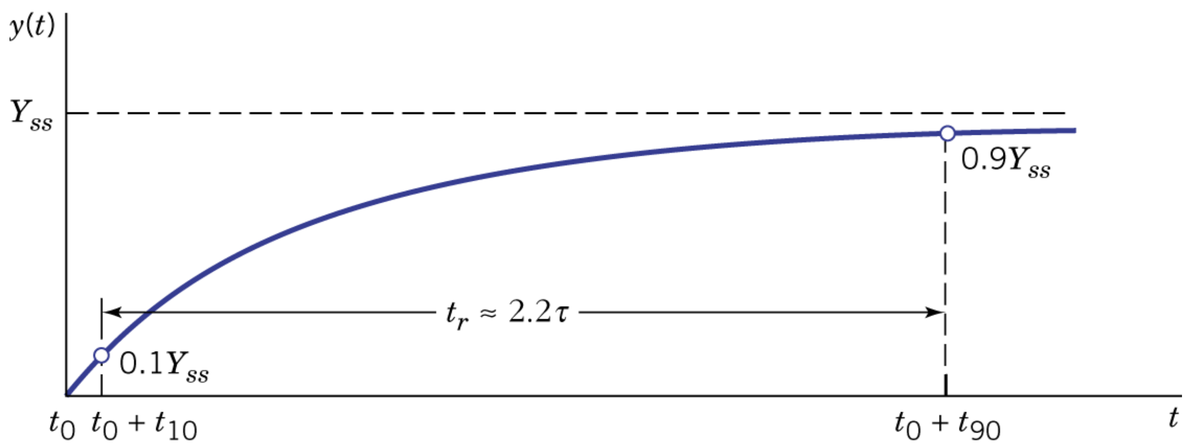
$$t = L / R_{eq}$$

I_{SS} : steady state current

General First-Order Circuits



(a) Step response waveform



(b) Expand view

$$y(t) = Y_{SS}\left[1 - e^{-(t-t_0)/t}\right], t > t_0$$

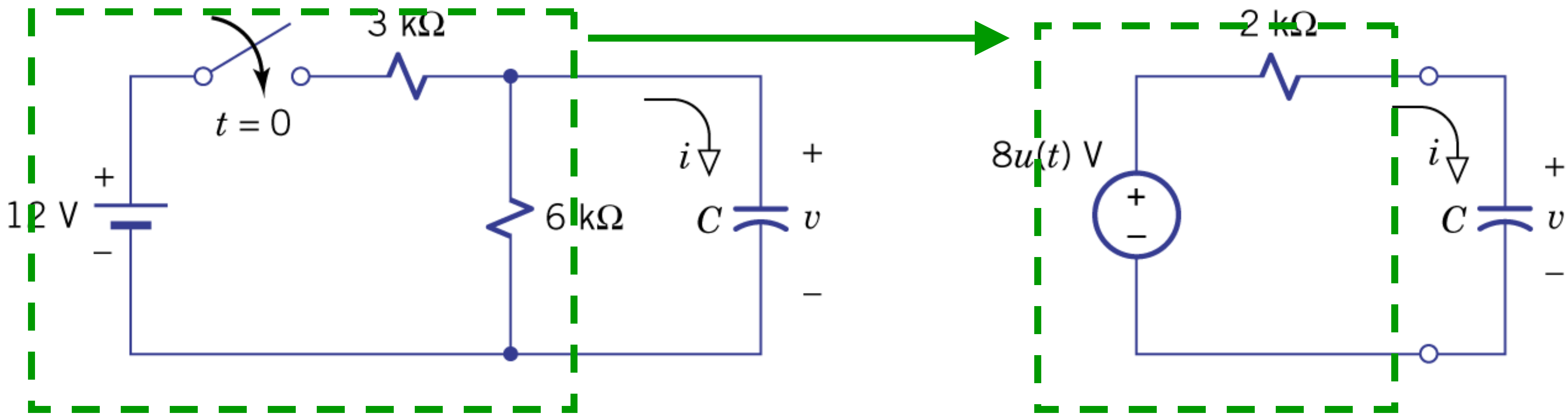
$$y(t) = Y_{SS}, \ t \rightarrow \infty \quad (\text{after } 5t)$$

$$\text{initial slope} = \frac{Y_{ss}}{t}$$

$5t$ to reach steady state

rise time (10% - 90%) $\approx 2.2t$

Example: Step Response of an RC Circuit



(a) Circuit for Example 9.2 (b) Equivalent circuit

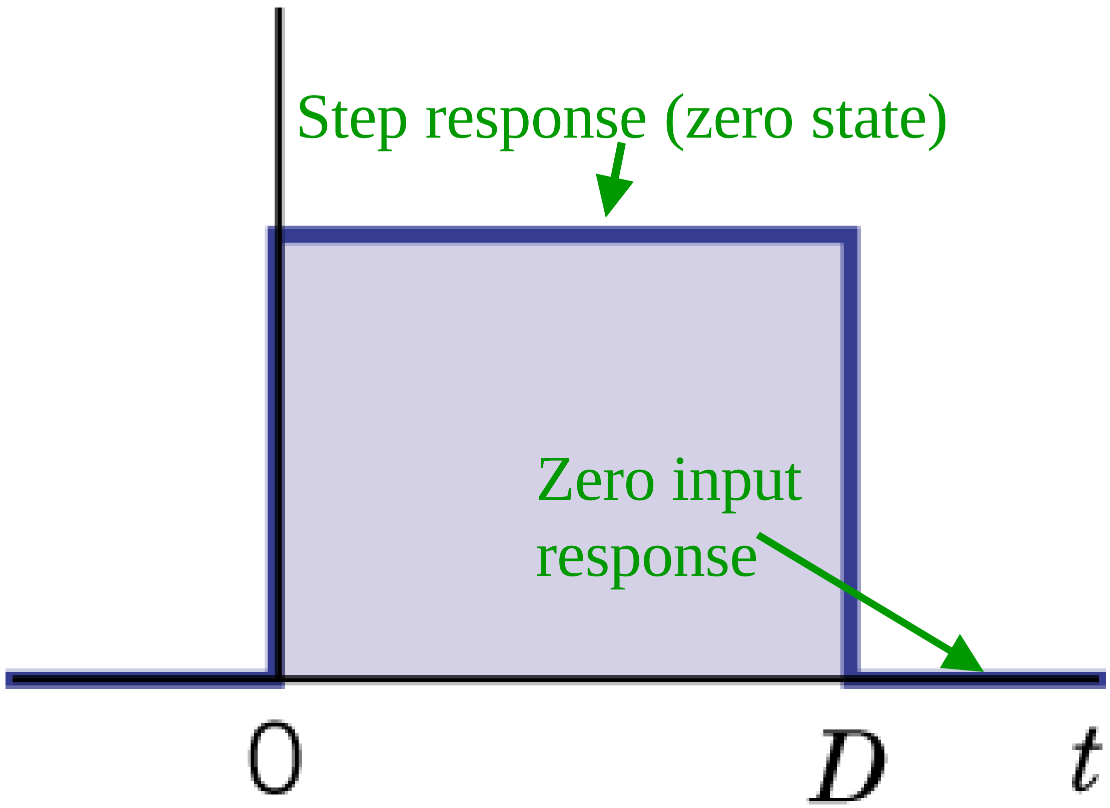
$$t = R_{eq} \cdot C = 0.1s \qquad 5t = 0.5s$$

$$v(t) = 8(1 - e^{-10t})V, \; t > 0 \qquad v(5t) = 8(1 - e^{-5}) \approx 8$$

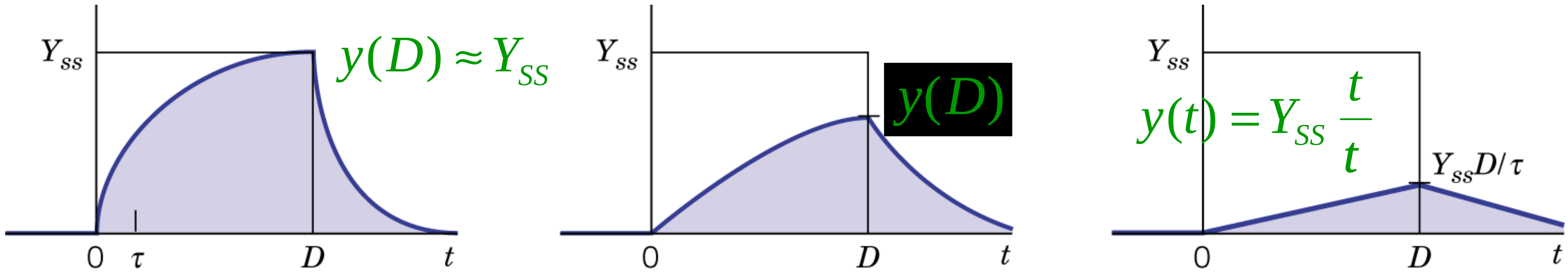
$$i(t) = 4e^{-10t} mA, \; t > 0 \qquad i(5t) = 4e^{-5} \approx 27mA \rightarrow 0$$

Pulse Response

- Pulse response: zero-sate response with a rectangular pulse excitation. Require initial value, steady state value and time constant.

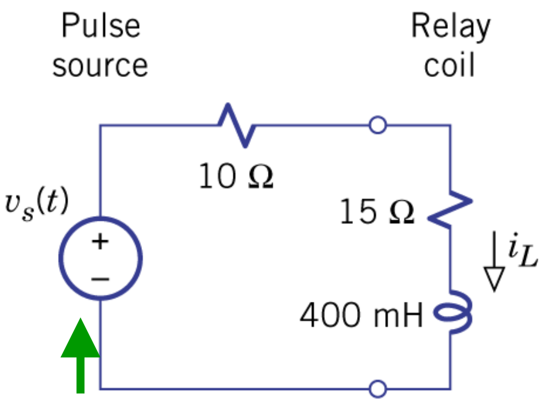


Pulse Response



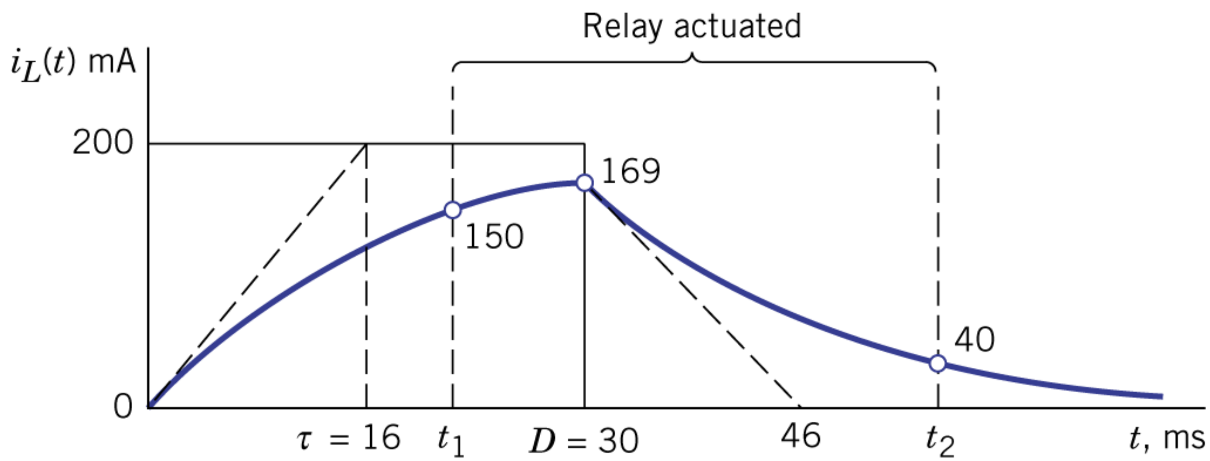
(a) $\tau \ll D$ (b) $\tau \approx D$ (c) $\tau \gg D$

Example 9.3: Analysis of a Relay Driver



5V, 30ms

(a) Relay driver



(b) Current waveform

$$t = L / R_{eq} = 16ms$$

$$\text{Norton}: \; I_{SS} = 5/25 = 200mA$$

$$i_L(t) = 200(1 - e^{-t/16})mA, \quad 0 < t \le 30ms$$

$$i_L(22.2ms) = 150mA$$

$$i_L(30ms) = 169mA$$

$$i_L(t) = 169e^{-(t-30)/16}mA, t > 30ms$$

$$i_L(53.1ms) = 40mA$$

total actuate time

$$= 53.1 - 22.2 = 30.9\text{ms}$$

Switched DC Transients

- Switched DC transients: the source switches from one constant to another constant.

Choose a state variable (v_C or i_L)

Find initial value $Y_0 = y(t_0^+) = y(t_0^-)$

Find steady state value Y_{SS}

Find time constant t (Thevenin/ Norton)

$$t y' + y = Y_{SS}, \; t > t_0$$

$$y(t) = Y_{SS} + Ae^{-t/t} = Y_{SS} + (Y_0 - Y_{SS})e^{-(t-t_0)/t}, \; t > t_0$$

Other circuit variables are similarly derived from state variables (may not have continuity)

General DC Response

$$y(t) = Y_{SS} + (Y_0 - Y_{SS})e^{-(t-t_0)/t}$$

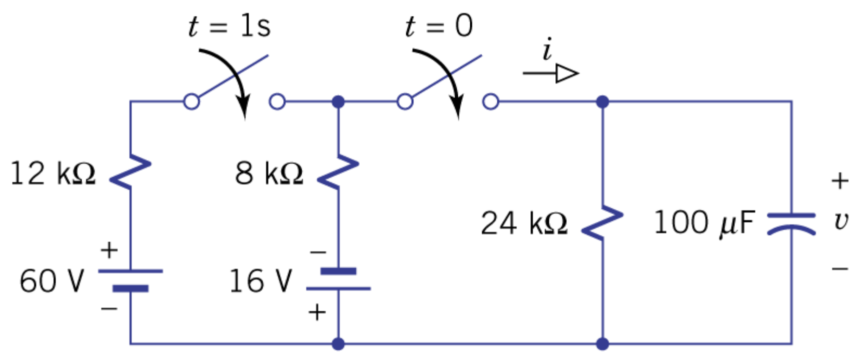
$$t = RC \text{ for series RC circuits}$$

$$t = \frac{L}{R} \text{ for parallel RL circuits}$$

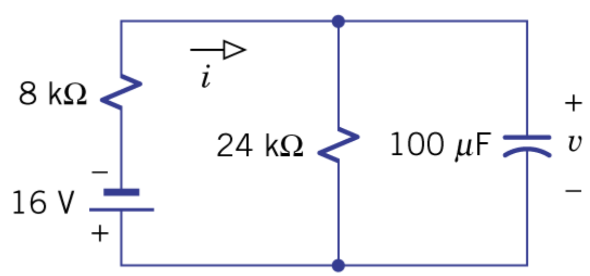
$$\text{Zero input response}: Y_{SS} = 0 \Rightarrow y(t) = Y_0 e^{-(t-t_0)/t}$$

$$\text{Step response}: Y_0 = 0 \Rightarrow y(t) = Y_{SS}\left(1 - e^{-(t-t_0)/t}\right)$$

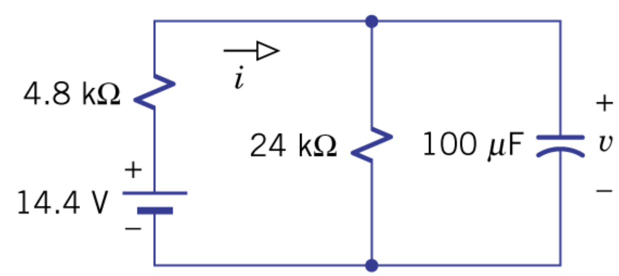
Example 9.4: Sequential Switched Transients



(a) Circuit with sequential switching



(b) Equivalent circuit for $0 < t \leq 1s$

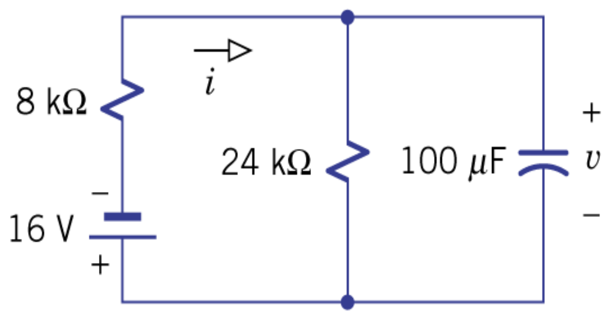


(c) Equivalent circuit for $t > 1s$

Part (I)

Part (II)

Example 9.4: Sequential Switched Transients (Part (I))



(b) Equivalent circuit for $0 < t \le 1s$

$$V_0 = v(0^+) = v(0^-)$$

$$I_0 = i\left(0^+\right) = -\frac{16}{8} = -2mA$$

$$V_{SS} = -16\frac{24}{8+24} = -12V$$

$$I_{SS} = -16\frac{1}{8+24} = -0.5mA$$

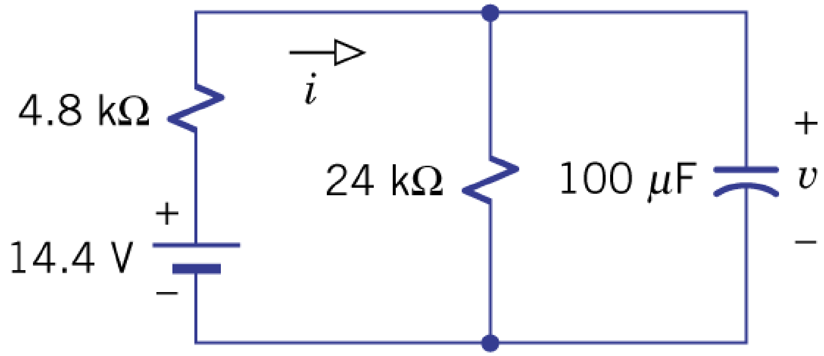
$$R_{eq} = 8 \,\|\, 24 = 6k\Omega$$

$$t = 6k \cdot 100m = 0.6s$$

$$v(t) = V_{SS} + (V_0 - V_{SS})e^{-t/t} = -12 + 12e^{-t/0.6}V$$

$$i(t) = I_{SS} + (I_0 - I_{SS})e^{-t/t} = -0.5 - 1.5e^{-t/0.6}mA$$

Example 9.4: Sequential Switched Transients (Part (II))



(c) Equivalent circuit for $t > 1s$

$$V_0 = v(1^+) = v(1^-) = \text{-}9.73V$$

$$I_0 = i\left(1^+\right) = 5.03mA$$

$$V_{SS} = 12V$$

$$I_{SS} = 0.5mA$$

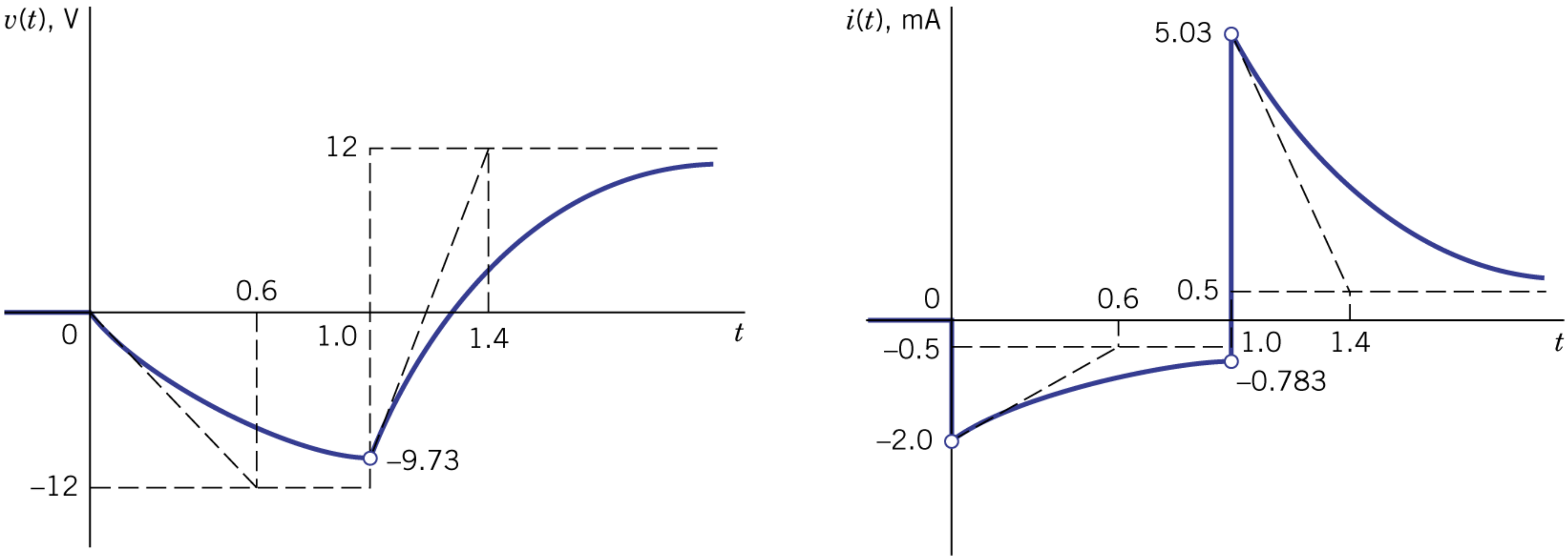
$$R_{eq} = 4k\Omega$$

$$t = 0.4s$$

$$v(t) = 12 - 21.73e^{-(t-1)/0.4}V$$

$$i(t) = 0.5 + 4.53 - 1.5e^{-(t-1)/0.4}mA$$

Example 9.4: Sequential Switched Transients



(d) Switched waveforms

Switched AC Transients

Switched AC Transients

- The parameter of an ac source undergoes an abrupt change.
- Analysis is the same as switched dc transients, except that the force response is different.

DC Response :

$$ty' + y = Y_{SS},\ t > t_0$$

$$y(t) = Y_{SS} + (Y_0 - Y_{SS})e^{-(t-t_0)/t},\ t > t_0$$

AC Response :

$$Y_{SS} \rightarrow y_F(t) = Y_m \cos(wt + f)$$

Use phasor analysis to obtain $y_F(t)$

Switched AC Transients

AC Response :

$$y(t) = y_F(t) + Ae^{-t/t}, \; t > t_0$$

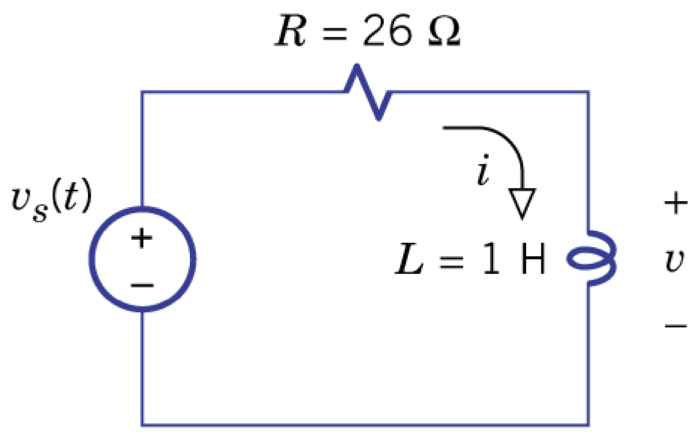
$$y(t_0^+) = y_F(t_0^+) + Ae^{-t_0/t} = Y_0$$

$$y(t) = y_F(t) + (Y_0 - y_F(t_0^+))e^{-(t-t_0)/t}, \; t > t_0$$

$y(t) \rightarrow$ steady state $(y_F(t))$ after $5t$

Use phasor analysis to obtain $y_F(t)$

Example 9.5: Transients in an AM (amplitude modulation) Radio Signal



(a) Circuit with switched ac source

$$v_S(t) = \begin{cases} 6\cos 15t, & t < 0 \\ 12\cos 15t, & t > 0 \end{cases}$$

Phasor analysis for t < 0 :

$$\underline{I} = \frac{6}{26 + j15} = 0.2A\angle -30^0$$

$$\Rightarrow i(t) = 0.2\cos(15t - 30^0)$$

$$\underline{V} = j15\underline{I} = 3V\angle 60^0$$

$$\Rightarrow v(t) = 3\cos(15t + 60^0)$$

$$I_0 = i(0^+) = i(0^-) = 0.2\cos(-30^0) = 0.173A$$

$$V_0 = v(0^+) = v_S(0^+) - 26 \cdot i(0^+) = 7.5V$$

Example 9.5: (Cont.)

for t > 0

$$i_F(t) = 0.4\cos(15t - 30^0)$$

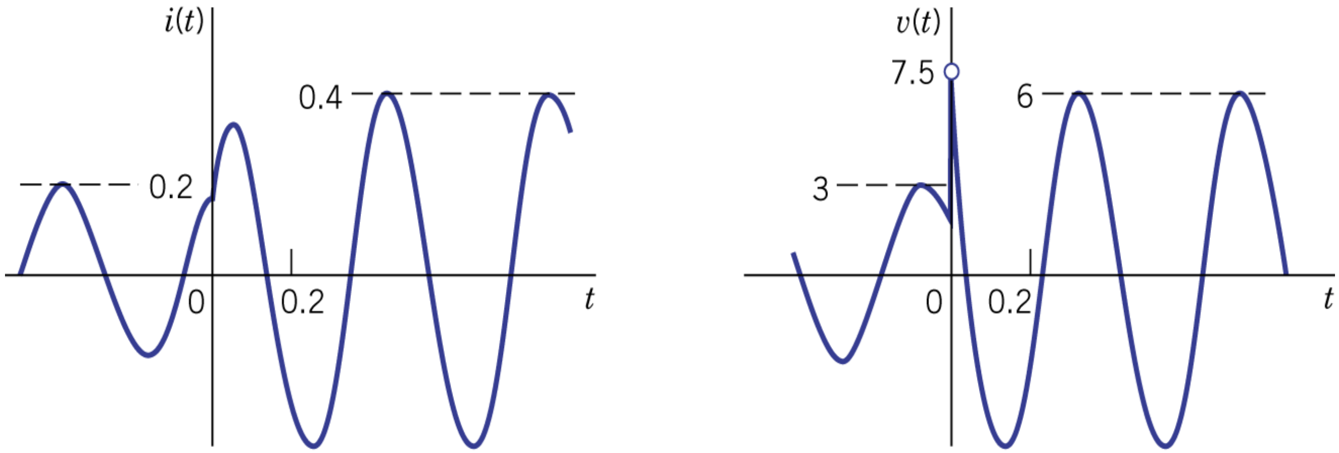
$$v_F(t) = 6\cos(15t + 60^0)$$

$$t = \frac{1}{26}s$$

$$i(t) = 0.4\cos(15t - 30^0) - 0.173e^{-26t}A$$

$$v(t) = 6\cos(15t + 60^0) + 4.5e^{-26t}V$$

$$5t \approx \left.\frac{T_0}{2}\right|_{w_0=15}$$



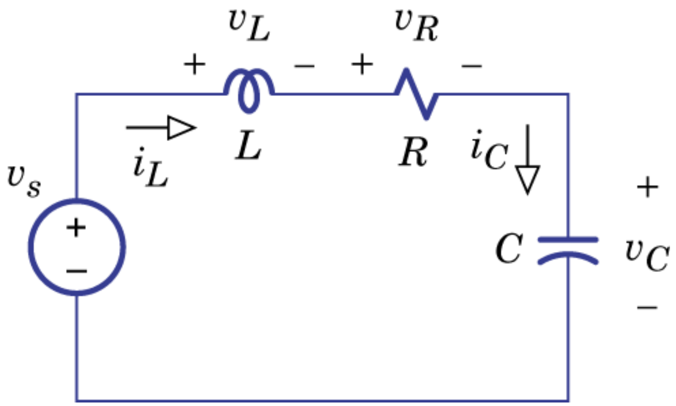
(b) Waveforms

Second-Order Natural Response

Second-Order Circuits

- A second-order circuit contains two independent energy storage elements (i.e., inductors or capacitors).
- First-order transient is characterized by decaying exponentials. Second order natural response includes overdamped, underdamped and critically damped behaviors.
- Capacitor voltages and inductor currents are state variables.

Series LRC Circuit



State vavriables: i_L, v_C.

$$\text{Example}: v_L = L\frac{di_L}{dt} = LC\frac{d^2 v_C}{dt^2}$$

$$\text{KVL}: \ v_S = v_L + v_R + v_C$$

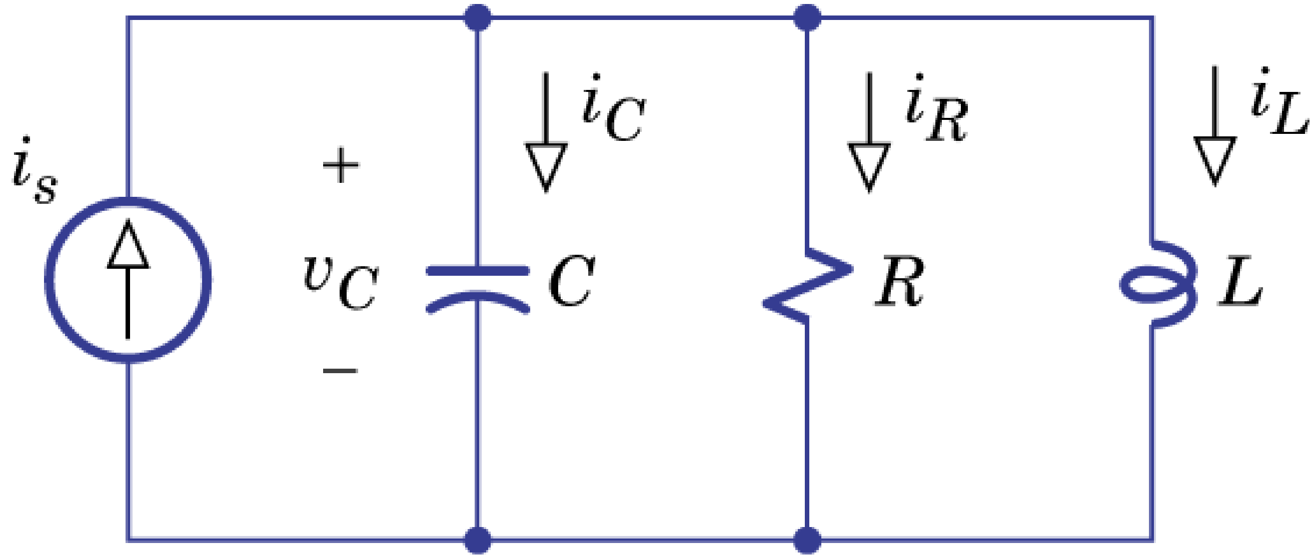
$$v_C'' + \frac{R}{L}v_C' + \frac{1}{LC}v_C = \frac{1}{LC}v_S$$

$$\text{or} \quad i_L'' + \frac{R}{L}i_L' + \frac{1}{LC}i_L = v_S'$$

$$\text{or} \quad v_L'' + \frac{R}{L}v_L' + \frac{1}{LC}v_L = \frac{1}{LC}v_S''$$

$\Rightarrow$ All circuit variables share the same form of natural response

Parallel CRL Circuit



Based on KCL, we have

$$i_L'' + \frac{1}{RC} i_L' + \frac{1}{LC} i_L = \frac{1}{LC} i_S$$

$$\text{or} \quad v_C'' + \frac{1}{RC} v_C' + \frac{1}{LC} v_C = \frac{1}{C} i_S'$$

General Form of Second-Order Circuits

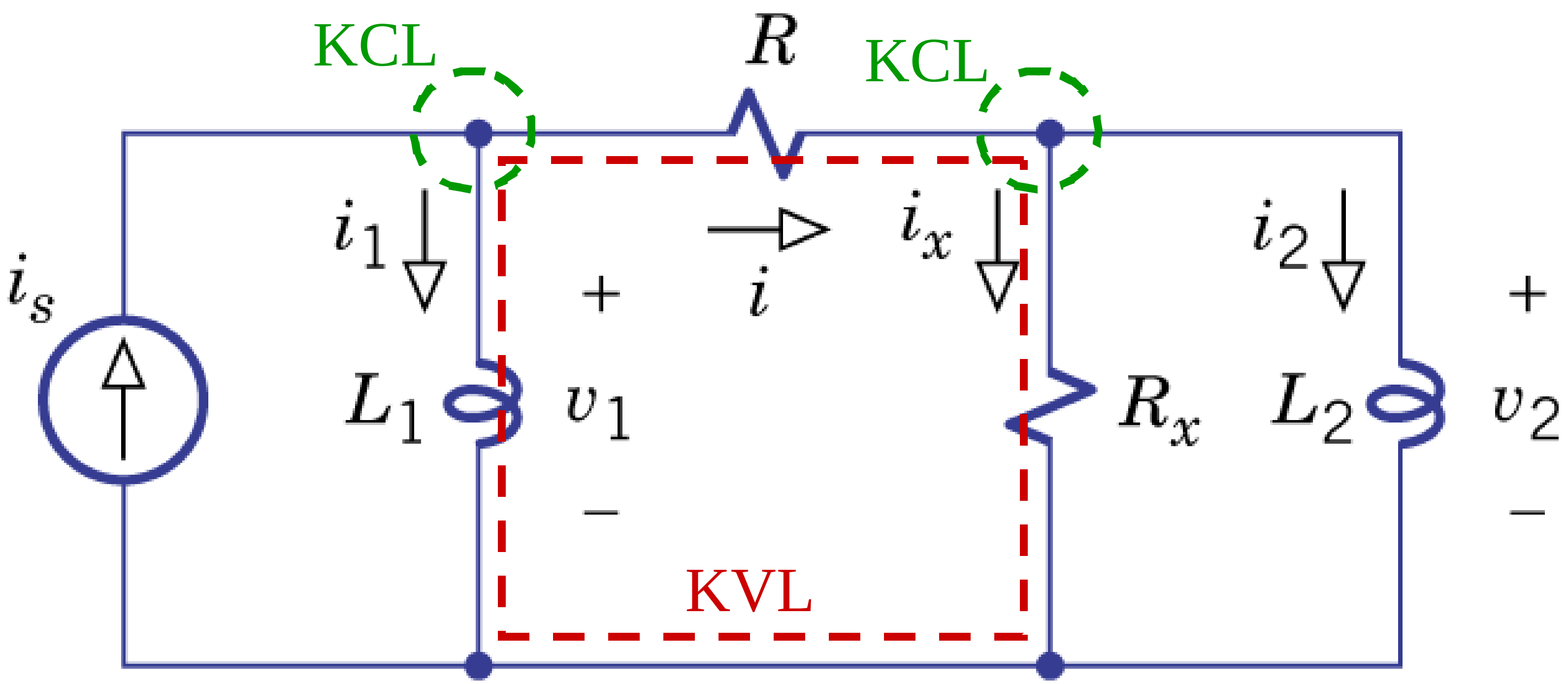
$$y'' + 2ay' + w_0^2 y = f(t)$$

$$\text{For parallel CRL}: a = \frac{1}{2RC}, w_0^2 = \frac{1}{LC}$$

$$\text{For series LRC}: a = \frac{R}{2L}, w_0^2 = \frac{1}{LC}$$

w_0 : resonant frequency

Example 9.6: Second-Order Circuit with Two Inductors



Pick i_1, i_2 as state variables.

Example 9.6: (Cont.)

$$\begin{cases} L_2 i_2' = R_x i_s - R_x i_1 - R_x i_2 \\ L_1 i_1' = (R + R_x) i_s - (R + R_x) i_1 - R_x i_2 \end{cases}$$

In matrix form (state equation) :

$$\begin{bmatrix} i_1' \\ i_2' \end{bmatrix} = \begin{bmatrix} -\dfrac{(R+R_x)}{L_1} & -\dfrac{R_x}{L_1} \\ -\dfrac{R_x}{L_2} & -\dfrac{R_x}{L_2} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \begin{bmatrix} \dfrac{(R+R_x)}{L_1} i_s \\ \dfrac{R_x}{L_2} i_s \end{bmatrix}$$

$$\left(\underline{i}' = A\underline{i} + \underline{I}_S \right)$$

Can be generalized to *n*-th order circuits.

Example 9.6: (Cont.)

In the form of an n-th order differential equation :

$$i_2'' + \left(\frac{R_x}{L_2} + \frac{R + R_x}{L_1} \right) i_2' + \frac{RR_x}{L_1 L_2} i_2 = \frac{R_x}{L_2} i_s'$$

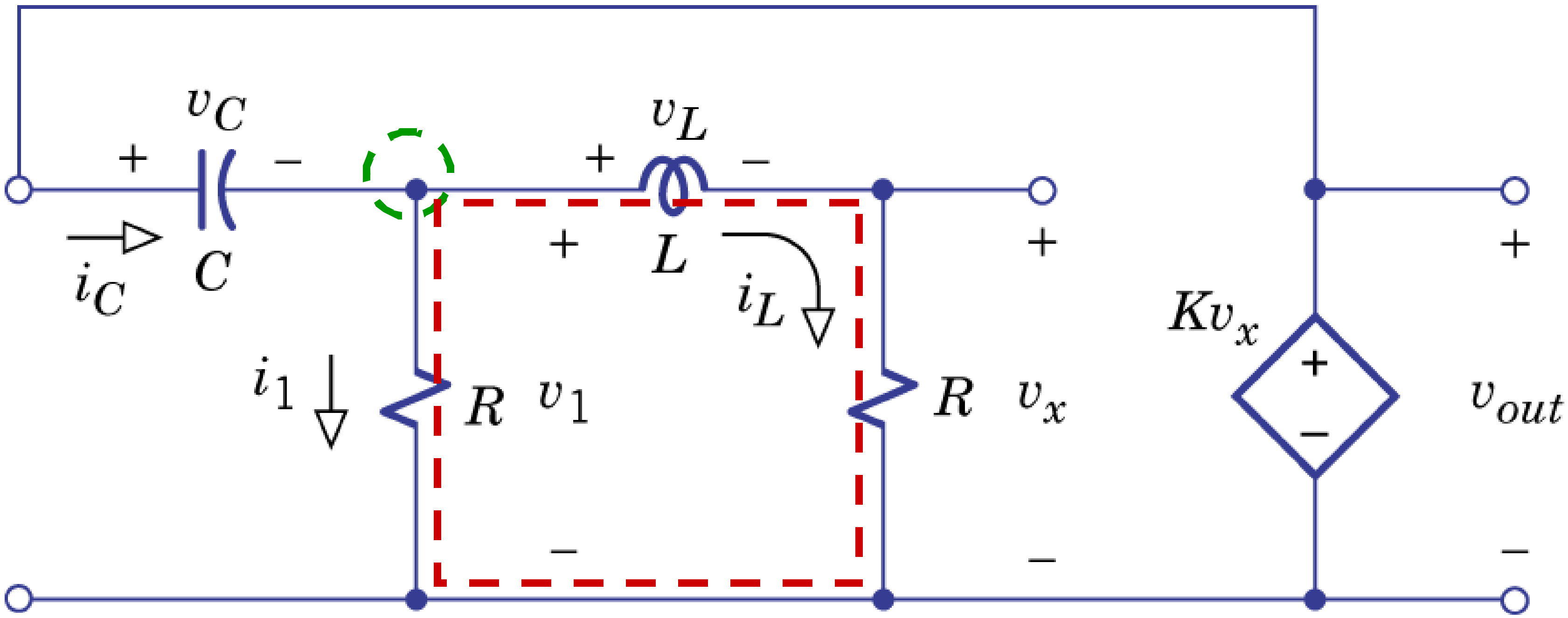
$$\Rightarrow i_2'' + 2a i_2' + w_0^2 i_2 = \frac{R_x}{L_2} i'$$

if $R \to \infty$, then $(L_1 + L_2) i_2' + R i_2 = L_1 i_s'$

$i_2 = i = i_1 - i_s$

First order circuit, energy stored in L_1 and L_2 are related.

Example 9.7: Second-Order with a Controlled Source (Phase-Shift Oscillator)



$$LCi_L'' + (RC - KRC + L/R)i_L' + 2i_L = 0$$

$$i_L = \frac{v_x}{R} = \frac{v_{out}}{KR}$$

$$v_{out}'' + \underbrace{\frac{R}{L}\left(\frac{L}{R^2C} + 1 - K\right)}_{2a} v_{out}' + \underbrace{\frac{2}{LC}}_{w_0^2} v_{out} = 0$$

Second-Order Natural Response

$$y_N'' + 2a y_N' + w_0^2 y_N = 0$$

$$s^2 + 2as + w_0^2 = 0$$

p_1, p_2 : roots, characteristic values,

natural frequencies, eigen values,...etc.

$$p_1, p_2 = -a \pm \sqrt{a^2 - w_0^2}$$

$$y_N(t) = A_1 e^{p_1 t} + A_2 e^{p_2 t} \quad (p_1 \neq p_2)$$

A_1, A_2 are determined by 2 initial conditins.

Second-Order Natural Response

$$p_1, p_2 = -a \pm \sqrt{a^2 - w_0^2}$$

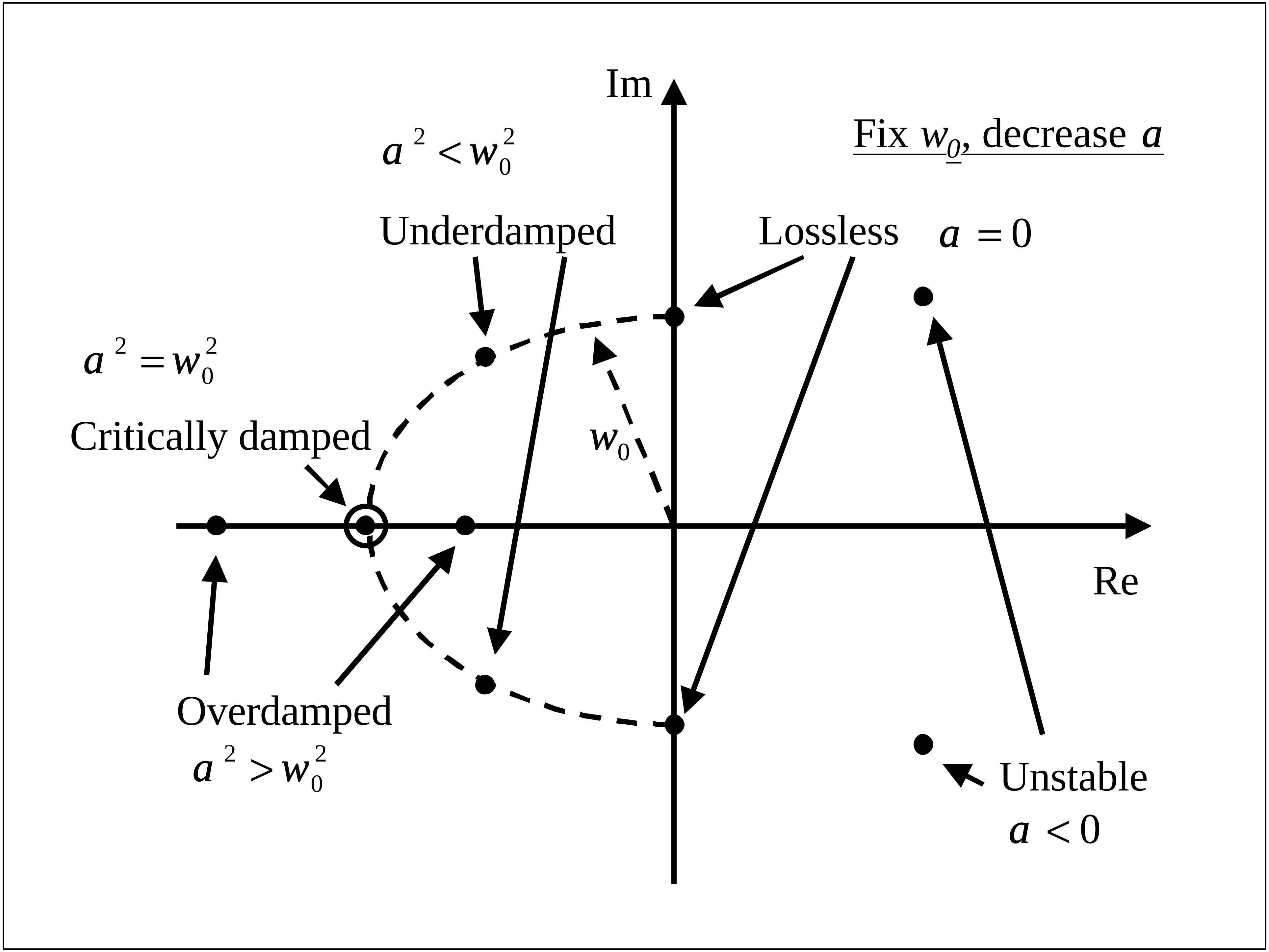
overdamped : $a^2 > w_0^2$ (distinct real roots)

underdamped : $a^2 < w_0^2$ (complex conjugate roots)

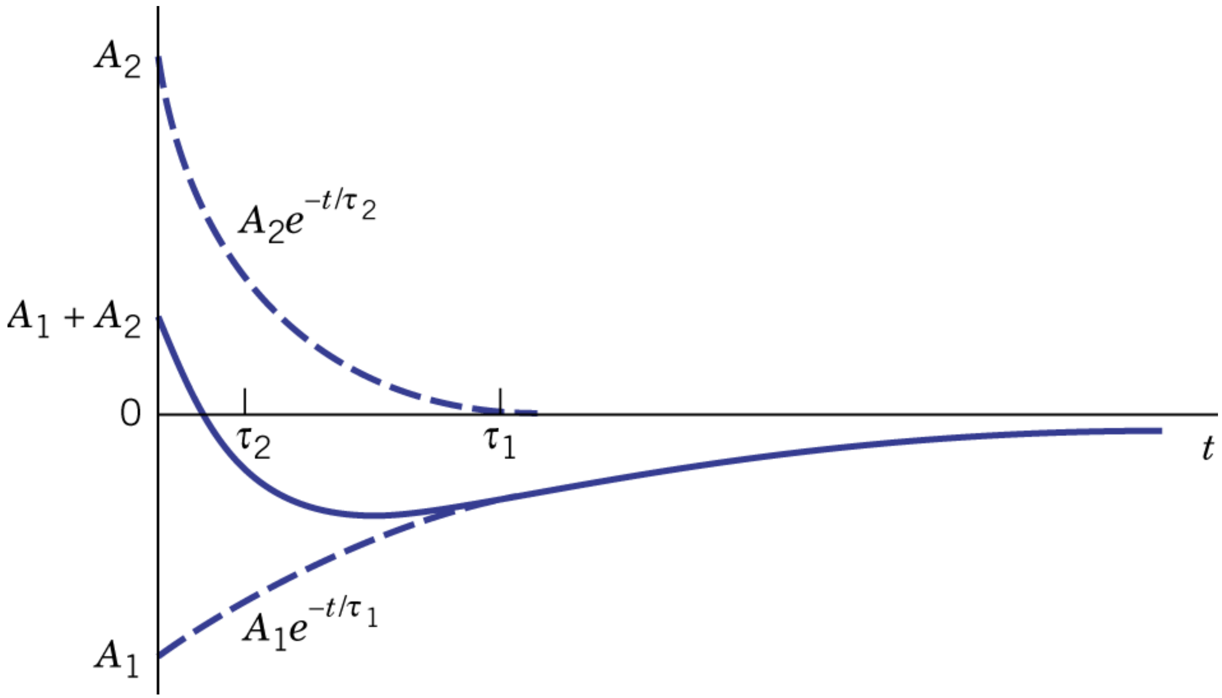
critically damped : $a^2 = w_0^2$ (repeated roots)

In some books :

lossless : $a = 0$ (imaginary roots)


Im
Fix w_0, decrease a
$a^2 < w_0^2$
Underdamped
Lossless
$a = 0$
$a^2 = w_0^2$
Critically damped
w_0
Re
Overdamped
$a^2 > w_0^2$
Unstable
$a < 0$

Overdamped Response



$$t_1 = -\frac{1}{p_1}, t_2 = -\frac{1}{p_2}$$

$$y_N(t) = A_1 e^{-t/t_1} + A_2 e^{-t/t_2}$$

$$a > 0 \Rightarrow \text{stable (i.e., } y_N(t) \to 0 \text{ as } t \to \infty)$$

Example 9.8: Natural Response of a Series LRC Circuit

$$i_L'' + \frac{R}{L} i_L' + \frac{1}{LC} i_L = 0$$

$$L = 0.1H, R = 14\Omega, C = \frac{1}{400} F$$

$$s^2 + 140s + 4000 = 0$$

$$p_1, p_2 = -40, -100$$

$$\tau_1, \tau_2 = 25ms, 10ms$$

$$i_L(t) = A_1 e^{-40t} + A_2 e^{-100t}$$

Underdamped Response

$$p_1 = -a + jw_d, \;\; p_2 = p_1^* = -a - jw_d$$

$$w_d = \sqrt{w_0^2 - a^2} : \text{ damped frequency}$$

$$a : \text{damping coefficient}$$

$$y_N(t) = e^{-at}(A_1 e^{jw_d t} + A_2 e^{-jw_d t}) \quad \text{(must be real)}$$

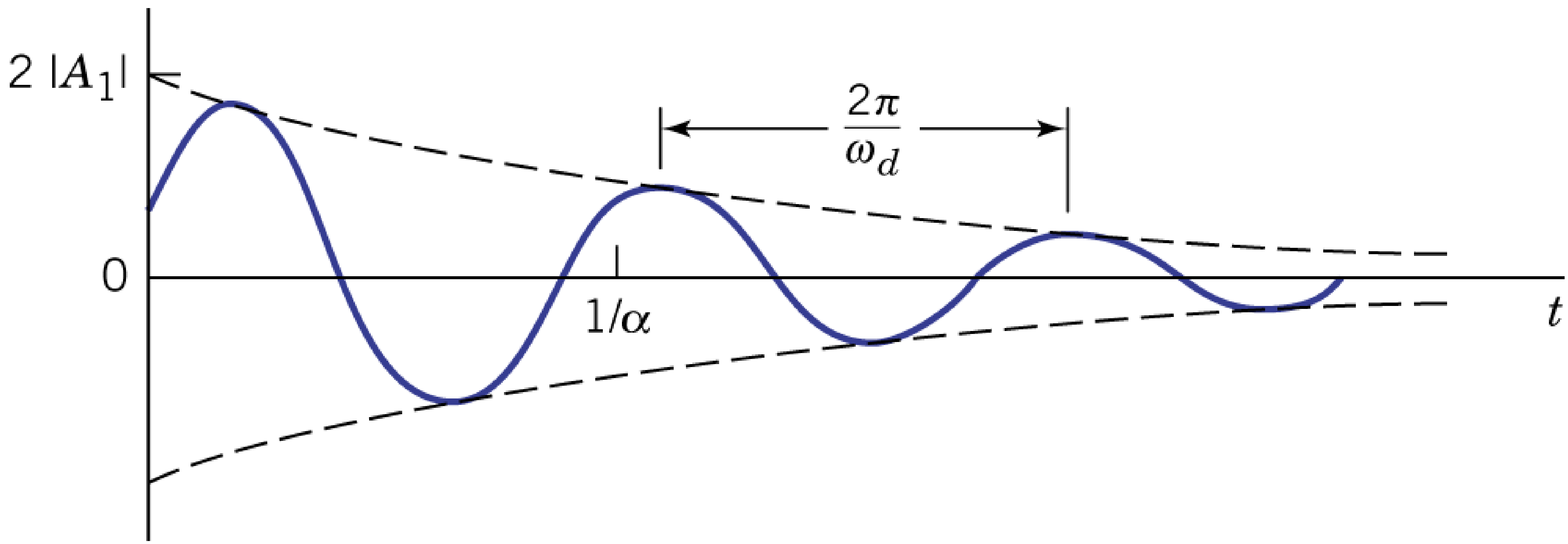
$$A_2 = A_1^*$$

$$y_N(t) = |A_1| e^{-at} \cos(w_d t + \angle A_1)$$

two initial conditions required for two unknowns

if $a = 0$ (lossless), $y_N(t) = 2|A_1| \cos(w_0 t + \angle A_1)$

Underdamped Response



Example 9.9: Natural Response of a Phase-Shift Oscillator

From example 9.7

Last indefinitely

$$a = 5000(2.5 - K), w_0^2 = 10^8$$

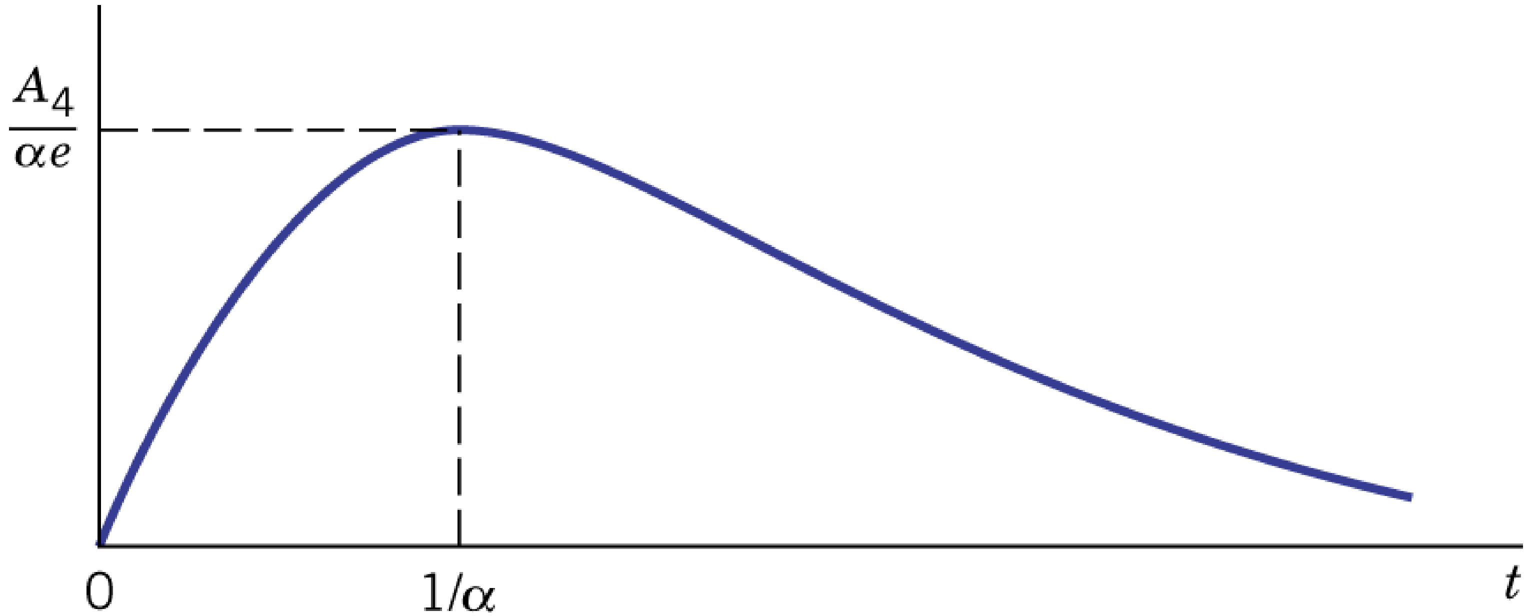
$$K = 2.5,\ a = 0,\ v_{out}(t) = 2|A_1|\cos(10000t + \angle A_1)$$

$$K = 2,\ a = 2500,\ w_d = 9680, v_{out}(t) = 2|A_1|e^{-2500t}\cos(9680t + \angle A_1)$$

$$K = 3,\ a = -2500,\ w_d = 9680, v_{out}(t) = 2|A_1|e^{2500t}\cos(9680t + \angle A_1)$$

Unstable

Critically Damped Response (Repeated Roots)



$$\text{solution}: y_N(t) = a(t)e^{-at}$$

$$\Rightarrow a(t) = A_3 + A_4 t$$

$$y_N(t) = A_3 e^{-at} + A_4 t e^{-at}$$

Please refer to Table 9.1.

Second-Order Transients

Second-Order Transients

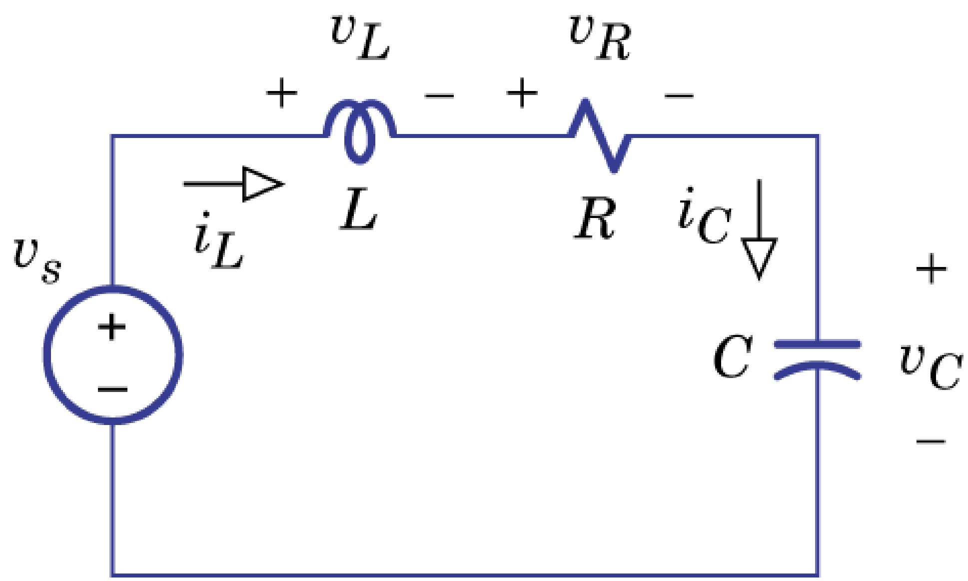
- Two initial conditions are required for second-order circuits.

Continuity for state variables: $v_C(0^+) = v_C(0^-), i_L(0^+) = i_L(0^-)$

If $v_C(0^+)$ and $i_L(0^+)$ are known, $v'_C(0^+) = \dfrac{i_C(0^+)}{C}, i'_L(0^+) = \dfrac{v_L(0^+)}{L}$

Initial slope

Example 9.10 Calculating Initial Conditions



$$v_S = \begin{cases} V_1, t < 0 \\ V_2, t > 0 \end{cases}$$

$$v_L = 0, i_C = 0, t < 0$$

$$\Rightarrow i_L(0^+) = i_L(0^-) = 0, v_C(0^+) = v_C(0^-) = V_1$$

Example 9.10 Calculating Initial Conditions

$$i_L(0^+) = \frac{1}{L} v_L(0^+) = \frac{1}{L}\left(V_2 - Ri_L(0^+) - v_C(0^+)\right) = \frac{1}{L}(V_2 - V_1)$$

$$v'_C(0^+) = \frac{1}{C} i_C(0^+) = \frac{1}{C} i_L(0^+) = 0$$

Then

$$v'_L(0^+) = -Ri'_L(0^+) - v'_C(0^+) = \frac{R(V_2 - V_1)}{L}$$

Switched DC Transients

- Switched DC transients for the three types of second-order circuits (overdamped, underdamped and critically damped).
- Similar to first order circuits:

$$y'' + 2ay' + w_0^2 y = w_0^2 Y_{SS}, t > 0$$

$$y(t) = Y_{SS} + y_N(t), t > 0$$

Switched DC Transients: Overdamped

$$p_1, p_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$y_N(t) = A_1 e^{p_1 t} + A_2 e^{p_2 t}$$

$$y(t) = Y_{SS} + A_1 e^{p_1 t} + A_2 e^{p_2 t}, t > 0$$

$$y(0^+) = Y_{SS} + A_1 + A_2$$

$$y'(0^+) = p_1 A_1 + p_2 A_2$$

$$\begin{bmatrix} 1 & 1 \\ p_1 & p_2 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} y(0^+) - Y_{SS} \\ y'(0^+) \end{bmatrix}$$

Switched DC Transients: Underdamped

$$p_1, p_2 = -a \pm jw_d = -a \pm \sqrt{w_0^2 - a^2}$$

(Same as in the overdamped case)

$$y(t) = Y_{SS} + 2|A_1|e^{-at}\cos(w_d t + \angle A_1), t > 0$$

Switched DC Transients: Critically Damped

$$p_1 = p_2 = -a$$

$$y(t) = Y_{SS} + A_3 e^{-at} + A_4 t e^{-at}, t > 0$$

$$y'(t) = -aA_3 e^{-at} + A_4 e^{-at} - aA_4 t e^{-at}$$

$$\Rightarrow A_3 = y(0^+) - Y_{SS}$$

$$A_4 = y'(0^+) + aA_3$$

Example 9.11: Underdamped Zero-Input Response

$$\text{Series LRC with } \alpha = 25, \omega_0^2 = 6400, v_s(t) = \begin{cases} 30V, t < 0 \\ 0V, t > 0 \end{cases}$$

$$p_1, p_2 = -25 \pm j76$$

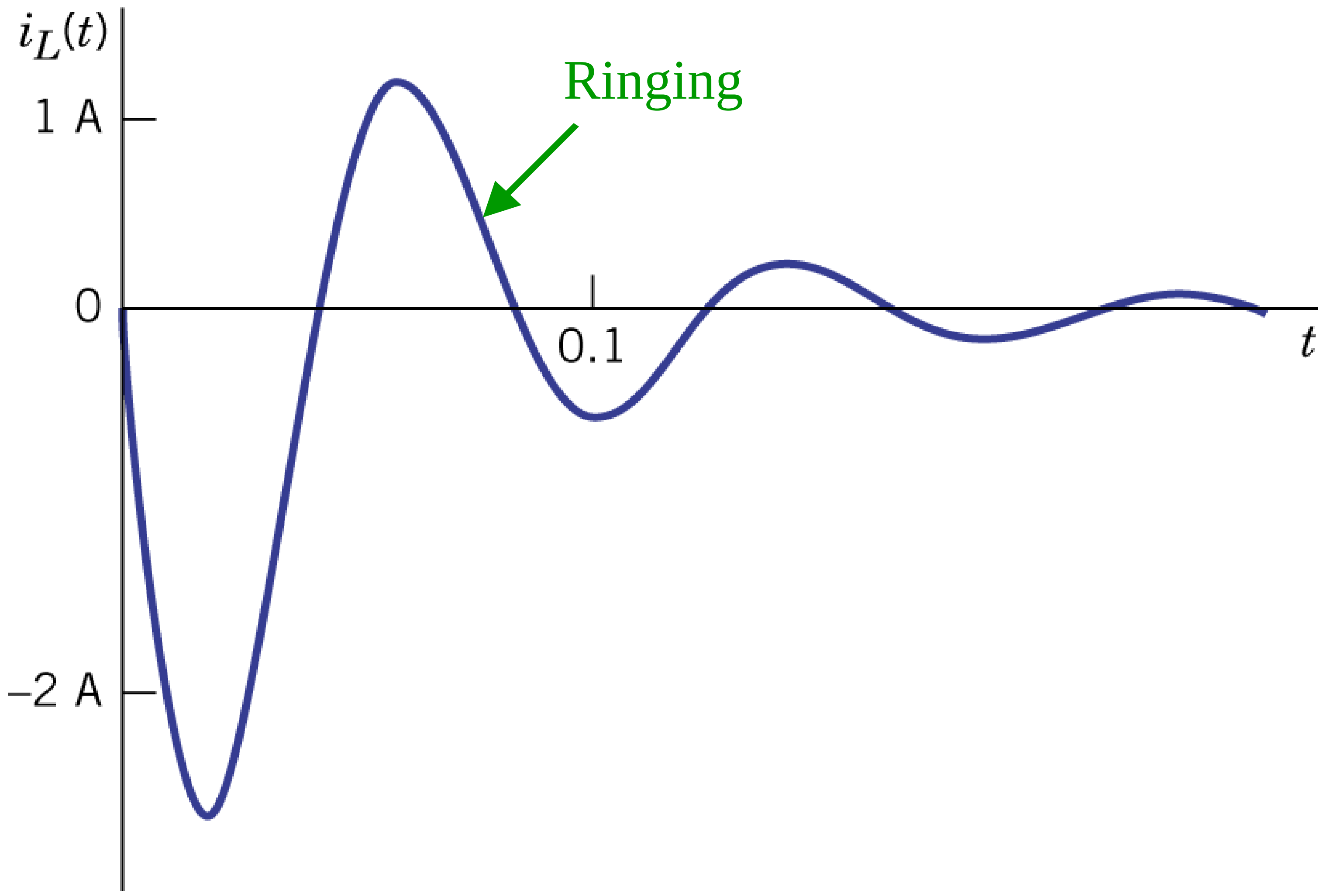
$$i_L(0^+) = 0, i_L'(0^+) = \frac{1}{L} v_L(0^+) = \frac{1}{L}(0 - 30) = -300$$

$$\begin{bmatrix} 1 & 1 \\ -25 + j76 & -25 - j76 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} i_L(0^+) - I_{SS} \\ i_L'(0^+) \end{bmatrix}$$

$$A_1 = 1.974A\angle 90^0$$

$$i_L(t) = 3.95e^{-25t}\cos(76t + 90^0)A, t > 0$$

Example 9.11: (Cont.)



Example 9.12: Step Response with Variable Damping (Series LRC)

$$v_s(t) = 30u(t) = \begin{cases} 0, t < 0 \\ 30, t > 0 \end{cases}$$

$$a = 5R, w_0^2 = 6400$$

$$V_1 = 0, V_2 = 30 = V_{SS}$$

$$v_C(0^+) = 0, v_C'(0^+) = 0$$

Maybe overdamped, underdamped and critically damped, depending on *R*.

Example 9.12: (Cont.)

$$\text{(i) overdamped}: R = 34, \alpha = 170$$

$$p_1, p_2 = -20, -320$$

$$\begin{bmatrix} 1 & 1 \\ -20 & -320 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} v_C(0^+) - V_{SS} \\ v'_C(0^+) \end{bmatrix} = \begin{bmatrix} -30 \\ 0 \end{bmatrix}$$

$$A_1 = -32, A_2 = 2$$

$$v_C(t) = 30 - 32e^{-20t} + 2e^{-320t} V, t > 0$$

Example 9.12: (Cont.)

(ii) overdamped : $R = 5, a = 25$

$$p_1, p_2 = -25 \pm j76$$

$$\begin{bmatrix} 1 & 1 \\ -25 + j76 & -25 - j76 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} -30 \\ 0 \end{bmatrix}$$

$$A_1 = 15.8 \angle 161.8^0$$

$$v_C(t) = 30 + 31.6 e^{-25t} \cos(76t + 161.8^0) V, t > 0$$

Example 9.12: (Cont.)

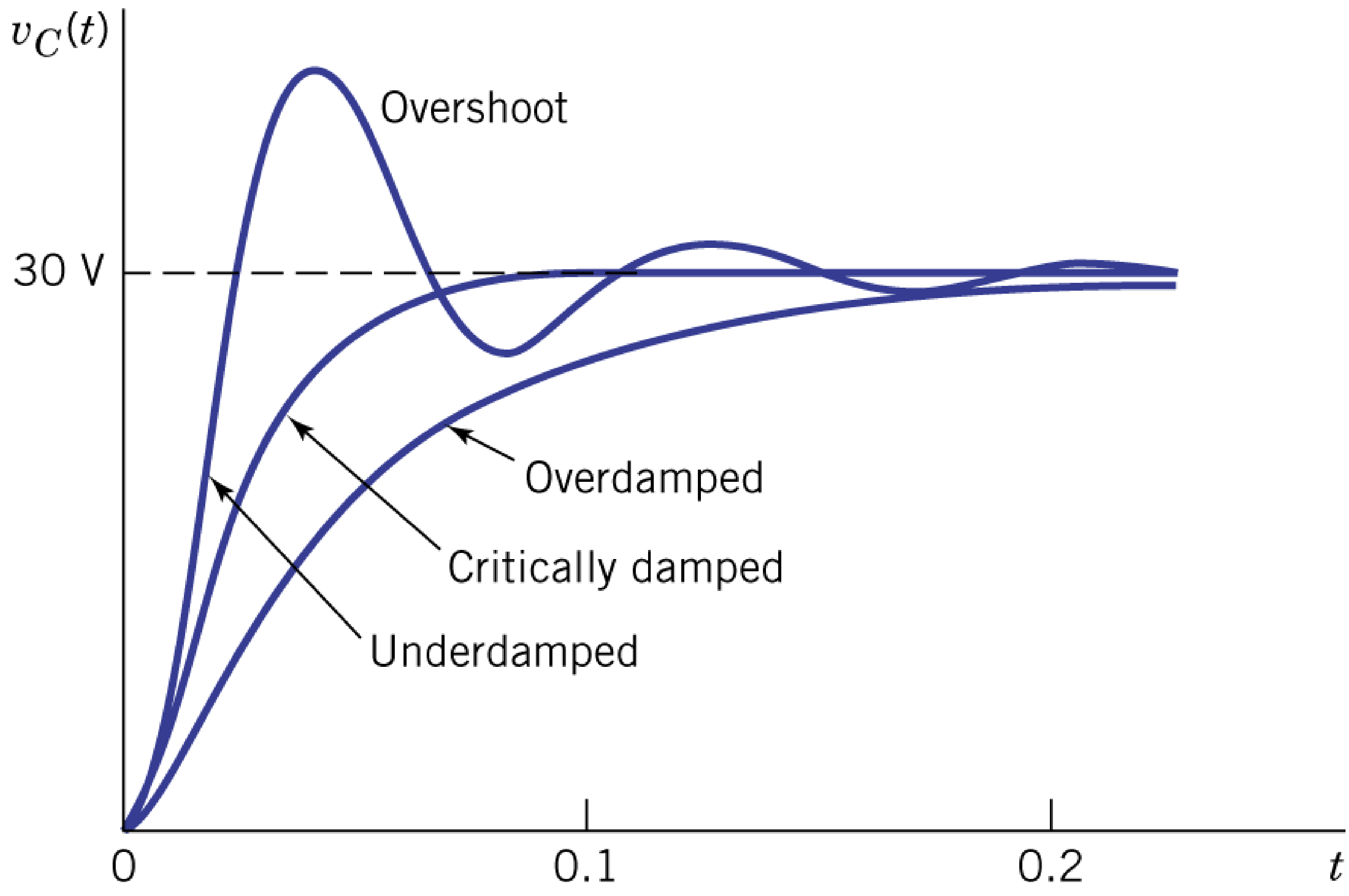
(iii) critically damped: $R = 16, a = 80$

$$A_3 = v_C(o^+) - V_{SS} = -30$$

$$A_4 = v_C'(o^+) + aA_3 = -2400$$

$$v_C(t) = 30 - 30e^{-80t} - 2400te^{-80t} V, t > 0$$

Example 9.12: (Cont.)



Chapter 9: Problem Set

- 1, 5, 7, 13, 16, 20, 21, 24, 30, 41, 44, 46, 52, 60.