Chapter 5: Energy Storage and Dynamic Circuits

Chapter 5: Outline

Energy Storage and Dynamic Circuits

(Time Varying Characteristics)

↓

Capacitor ⟷ Inductor (Dual)

Branch (Device) Equation in Differential Form

Power and Stored Energy

Memory and Continuity

Parallel and Series

Current Divider

↓

Dynamic Circuit (Differential Equation)

Natural Response (No Input, Homogeneous Solution)

Forced Response (Steady State, Particular Solution)

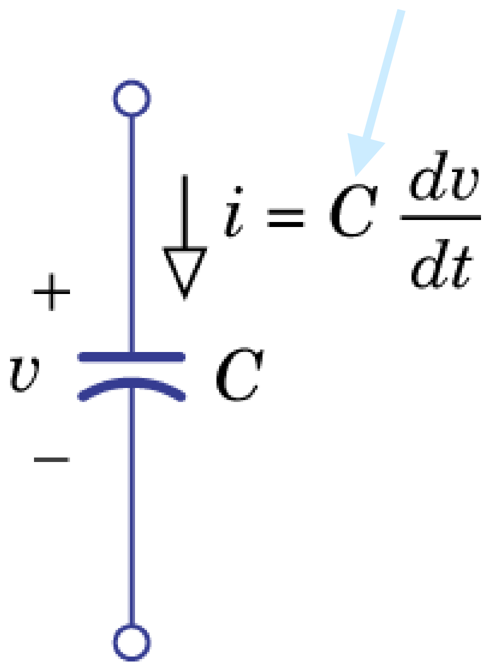
↓

Complete Response=Natural Response + Forced Response

Transient Response, Steady State Response

Capacitor (Brief)

Branch equation



instantaneous power : $p = vi = Cv\dfrac{dv}{dt}$

instantaneous stored energy : $w = \dfrac{1}{2}Cv^2$

<u>Electrical memory</u>

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(l)\,dl = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i(l)\,dl$$

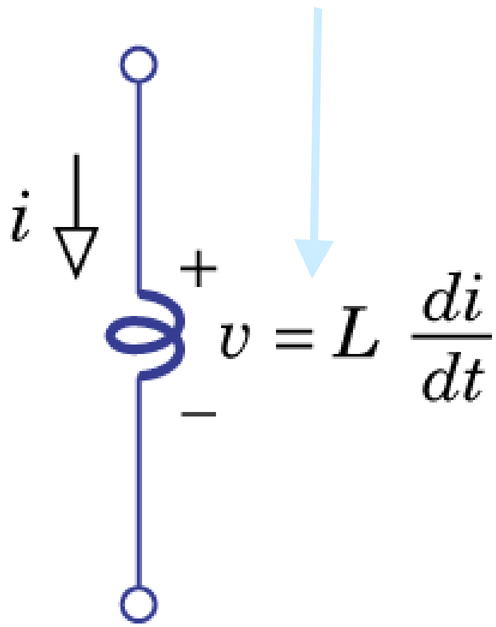
<u>Voltage continuity when the current is finite.</u>

$$v(t_j^+) = v(t_j^-) + \frac{1}{C}\int_{t_j^-}^{t_j^+} i(l)\,dl$$

<u>**v_c is the preferred variable.**</u>

Inductor (Brief)

Branch equation



instantaneous power : $p = vi = Li\frac{di}{dt}$

instantaneous stored energy : $w = \frac{1}{2}Li^2$

<u>Electrical memory</u>

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} v(l)dl = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v(l)dl$$

<u>Current continuity when the voltage is finite.</u>

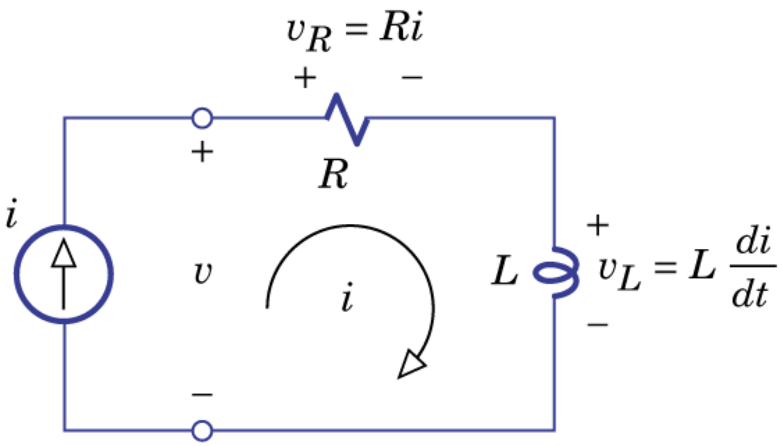
<u>**i_L is the preferred variable.**</u>

Dynamic Circuits

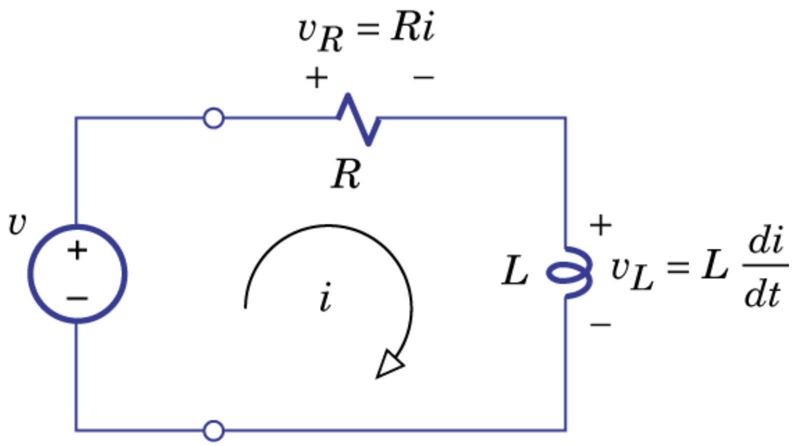
Dynamic Circuits

- A circuit is dynamic when currents or voltages are time-varying.
- Dynamic circuits are described by differential equations.
- Order of the circuit is determined by order of the differential equation.
- The differential equations are derived based on Kirchhoff’s laws and device (branch) equations.

First-Order Dynamic Circuits



(a) Series *RL* network with a current source



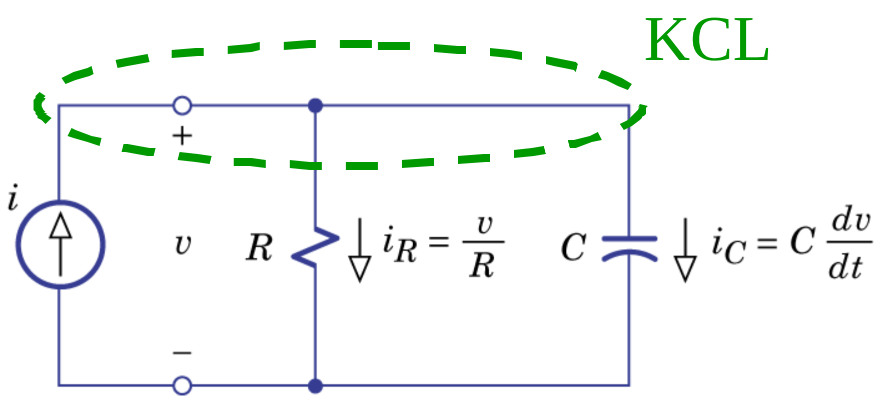
(b) Series *RL* network with a voltage source

direct form : $v = L\frac{di}{dt} + Ri$ indirect form : $v = L\frac{di}{dt} + Ri$

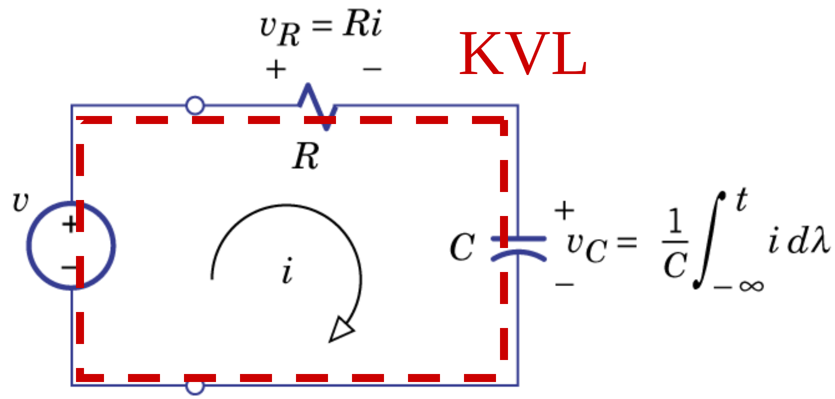
Generic form (differential): $a_1\frac{dy}{dt} + a_0 y = f(t)$

Forcing function

First-Order Circuits



(a) Parallel RC circuit



(b) Series RC circuit

$$c\frac{dv}{dt} + \frac{1}{R}v = i$$

$$v = Ri + \frac{1}{C}\int_{-\infty}^{t} i\,dl \quad \Rightarrow \quad R\frac{di}{dt} + \frac{1}{C}i = \frac{dv}{dt}$$

or $$RC\frac{dv_c}{dt} + v_c = v$$

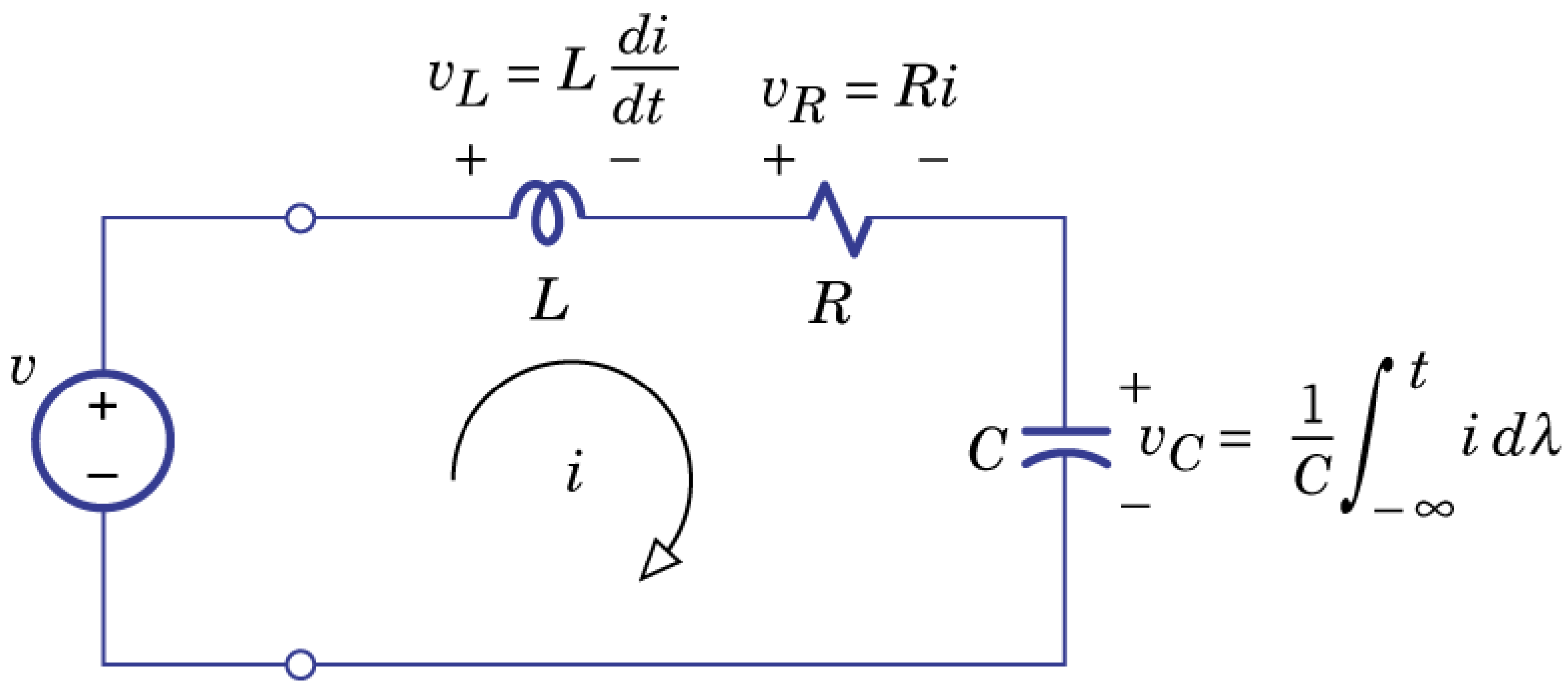
Generally, use v_C or i_L.

Second-Order Circuits

- Second-order circuits: circuits described by a second-order differential equation.

$$\text{Generic form}: a_2 \frac{d^2 y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y = f(t)$$

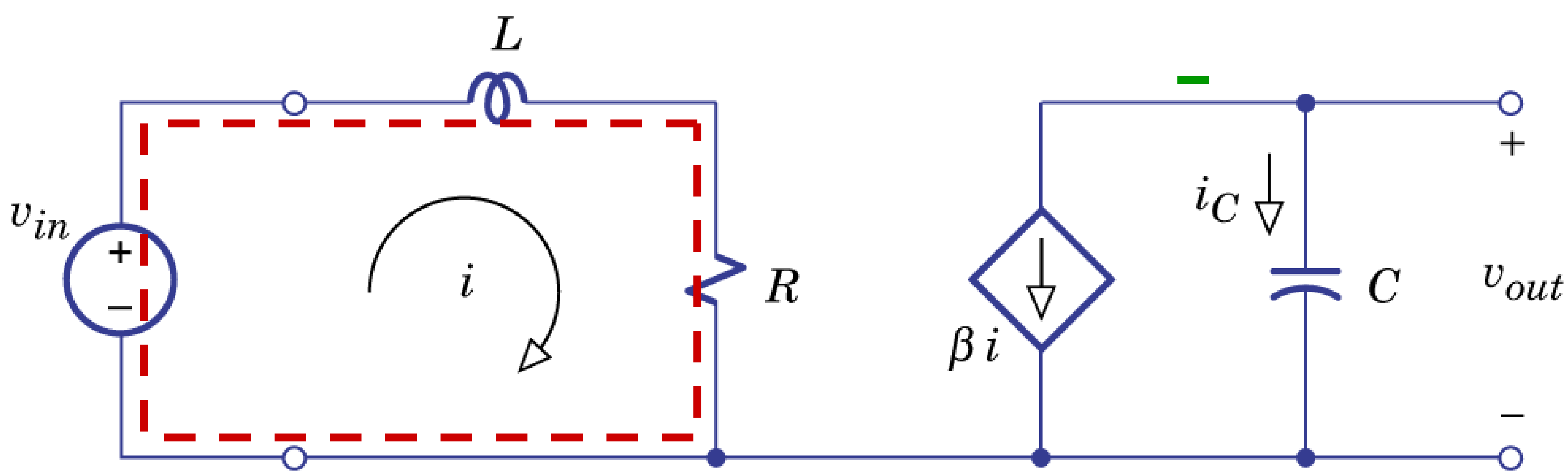
Second-Order Circuits



$$\text{KVL}: v = v_L + v_R + v_C$$

In differential form: $\dfrac{dv}{dt} = L\dfrac{d^2 i}{dt^2} + R\dfrac{di}{dt} + \dfrac{1}{C}i$

Example 5.8:
Second-Order Amplifier Circuit



$$v_{in} = L\frac{di}{dt} + Ri \qquad i = -\frac{C}{b}\frac{dv_{out}}{dt}$$

$$LC\frac{d^2 v_{out}}{dt^2} + RC\frac{dv_{out}}{dt} = -b v_{in}$$

Response:

- Natural, Forced
- Transient, Steady state
- many more,…

Natural Response

- Natural response $y_N(t)$ is the solution of the circuit equation with the forcing function set to zero. It is also known as the complementary solution.
- With the forcing function set to zero, the differential equation becomes homogeneous.
- A homogeneous differential equation can be solved using the characteristic equation along with the initial condition.
- First-order circuits require one initial condition. Second-order circuits require two initial conditions.

First-Order Circuits

$$a_1 \frac{dy_N}{dt} + a_0 y_N = 0$$

Characteristic equation : $a_1 s + a_0 = 0$

$$\Rightarrow s = -\frac{a_0}{a_1}$$

$$y_N(t) = Ae^{st} = Ae^{-\frac{a_0}{a_1}t}$$

A is determined by initial condition $y_N(0^+) = A = Y_0$

Second-Order Circuits

$$a_2 \frac{d^2 y_N}{dt^2} + a_1 \frac{dy_N}{dt} + a_0 y_N = 0$$

Characteristic equation : $a_2 s^2 + a_1 s + a_0 = 0$

$$\Rightarrow s = \boldsymbol{a}_1, \boldsymbol{a}_2 \quad \left(\frac{-a_1 \pm \sqrt{a_1^2 - 4a_2 a_0}}{2a_2} \right)$$

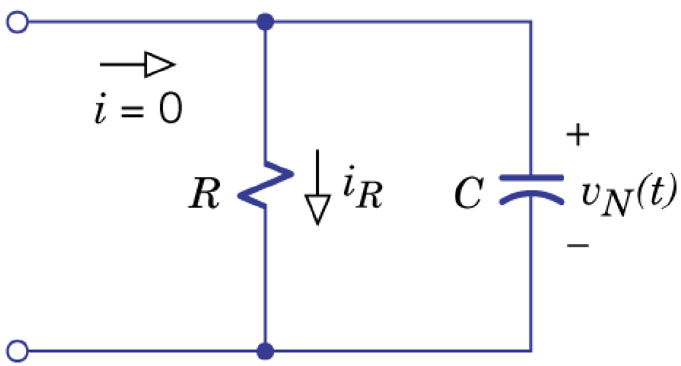
$$y_N(t) = A_1 e^{a_1 t} + A_2 e^{a_2 t}$$

A_1, A_2 are determined by two initial conditions $y_N(0^+)$ and $\frac{dy_N(0^+)}{dt}$

Stable Circuit

- A circuit is stable if the circuit variable $y_N(t) \rightarrow 0$, as $t \rightarrow \infty$.
- A circuit is exponentially stable if the circuit variable $y_N(t) \rightarrow 0$, as $t \rightarrow \infty$ in an exponential form.

Example 5.9: Capacitor Discharge



C=300mF
R=2MW
v(0⁻)=1000V

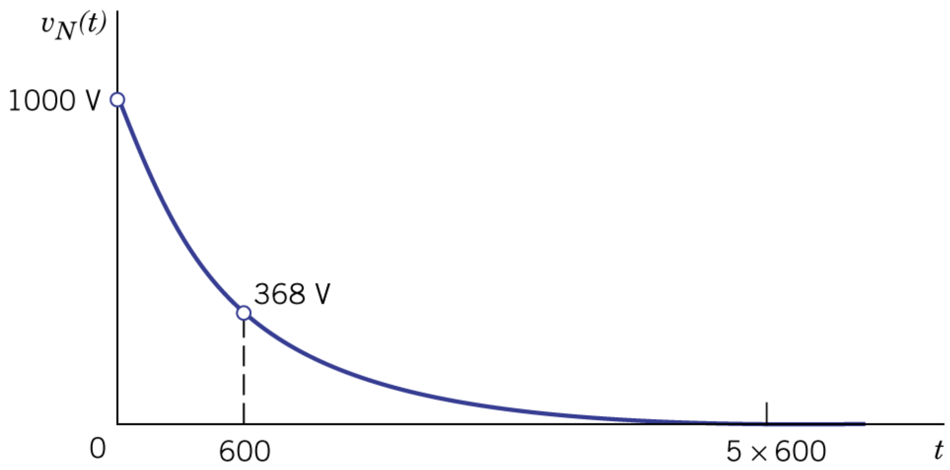
$$RC\frac{dv_N}{dt}+v_N=0$$

$$RCs+1=0 \Rightarrow s=-\frac{1}{RC}=-\frac{1}{600}$$

$$v_N(t)=Ae^{-t/600}$$

$$A=v_N(0^+)=v_N(0^-)=1000\ \text{(continuity)}$$

$$v_N(t)=1000e^{-\frac{t}{600}}V, t>0$$



Forced Response

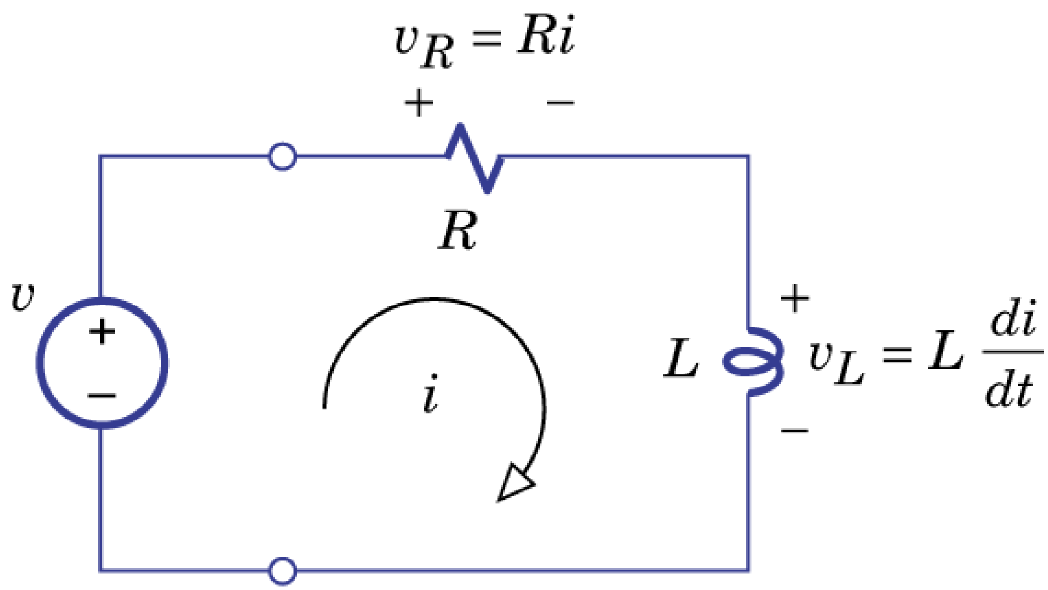
- Forced response $y_F(t)$ is the solution of the inhomogeneous differential equation (i.e., the forcing function is not zero), independent of any initial conditions. The solution is also known as the particular solution.
- Please refer to Table 5.3 for method of undetermined coefficients.

Table 5.3

TABLE 5.3 Selected Trial Solutions for Forced Response

$f(t)$	$y_F(t)$
k_0 (a constant)	K_0 (a constant)
$k_1 t$	$K_1 t + K_0$
$k_2 e^{at}$	$K_2 e^{at}$
$k_3 \cos \omega t + k_4 \sin \omega t$	$K_3 \cos \omega t + K_4 \sin \omega t$

Example 5.10 Sinusoidal Forced Response



(*b*) Series *RL* network with a voltage source

R=4W

L=0.1H

$V(t)$=25sin30t

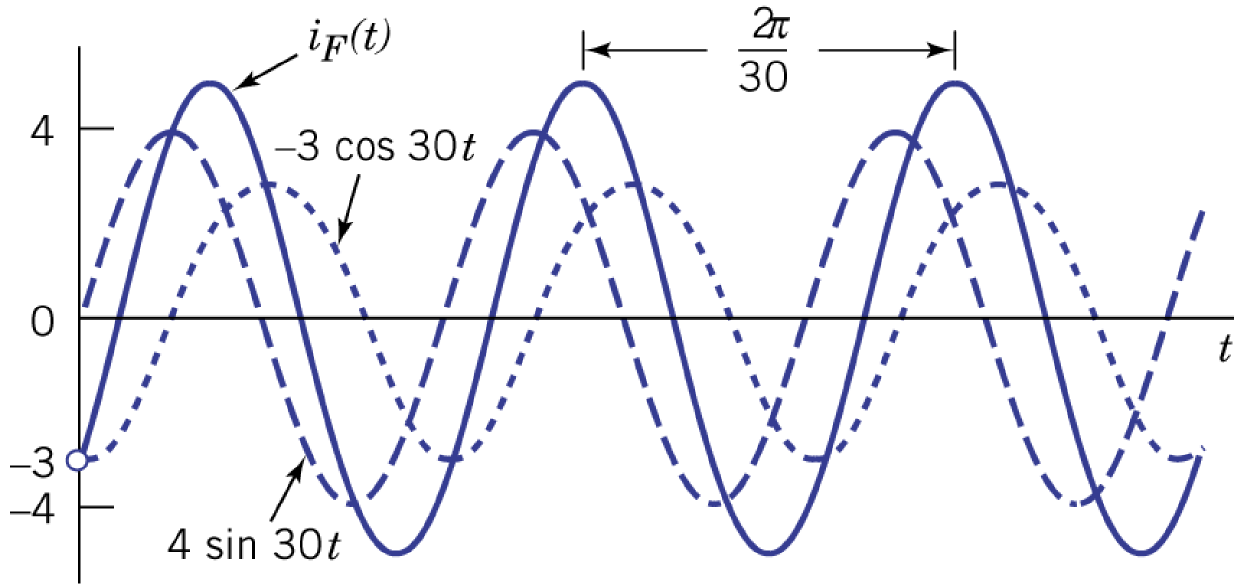
Example 5.10 Sinusoidal Forced Response

$$0.1\frac{di_F}{dt} + 4i_F = 25\sin 30t$$

$$i_F(t) = k_3\cos 30t + k_4\sin 30t$$

after substitution, we have $k_3 = -3, k_4 = 4$

$$i_F(t) = -3\cos 30t + 4\sin 30t \quad (\text{or } i_F(t) = A\cos(30t + q))$$



If the forcing function contains any term proportional to a component of the natural response (i.e., excitation of a natural frequency), then the term must be multiplied *t*.

Example 5.11: Exponential Forced Response

$$0.1\frac{di}{dt}+4i=v$$

$$\text{homogeneous solution}: 0.1s+4=0 \Rightarrow s=-40$$

$$v=10e^{-bt}$$

if $b=20$

$$i_F(t)=k_2e^{-20t}$$

$$k_2=5$$

$$i_F(t)=5e^{-20t}A$$

if $b=40$

$$i_F(t)=k_2te^{-40t}$$

$$k_2=100$$

$$i_F(t)=100te^{-40t}A$$

Complete Response

- Complete response is the sum of the natural response and the forced response, i.e., $y(t) = y_F(t) + y_N(t)$.
- The constants in $y_N(t)$ are evaluated from the initial conditions on with the complete response.
- For a stable circuit, $y(t)= y_F(t)$, as $t \rightarrow \infty$, since $y_N(t) \rightarrow 0$, as $t \rightarrow \infty$.
- The circuit is in the steady state if $y_N(t)$ is negligible compared to $y_F(t)$.
- Before arriving the steady state, the circuit is in the transient state.

Example 5.12: Complete Response Calculation (RL)

$$0.1\frac{di}{dt} + 4i = v, \quad i(t) = 0 \text{ for } t < 0$$

$$v(t) = 400\sin 280t, \text{ for } t > 0$$

$$\text{homogeneous solution}: i_N(t) = Ae^{-40t}$$

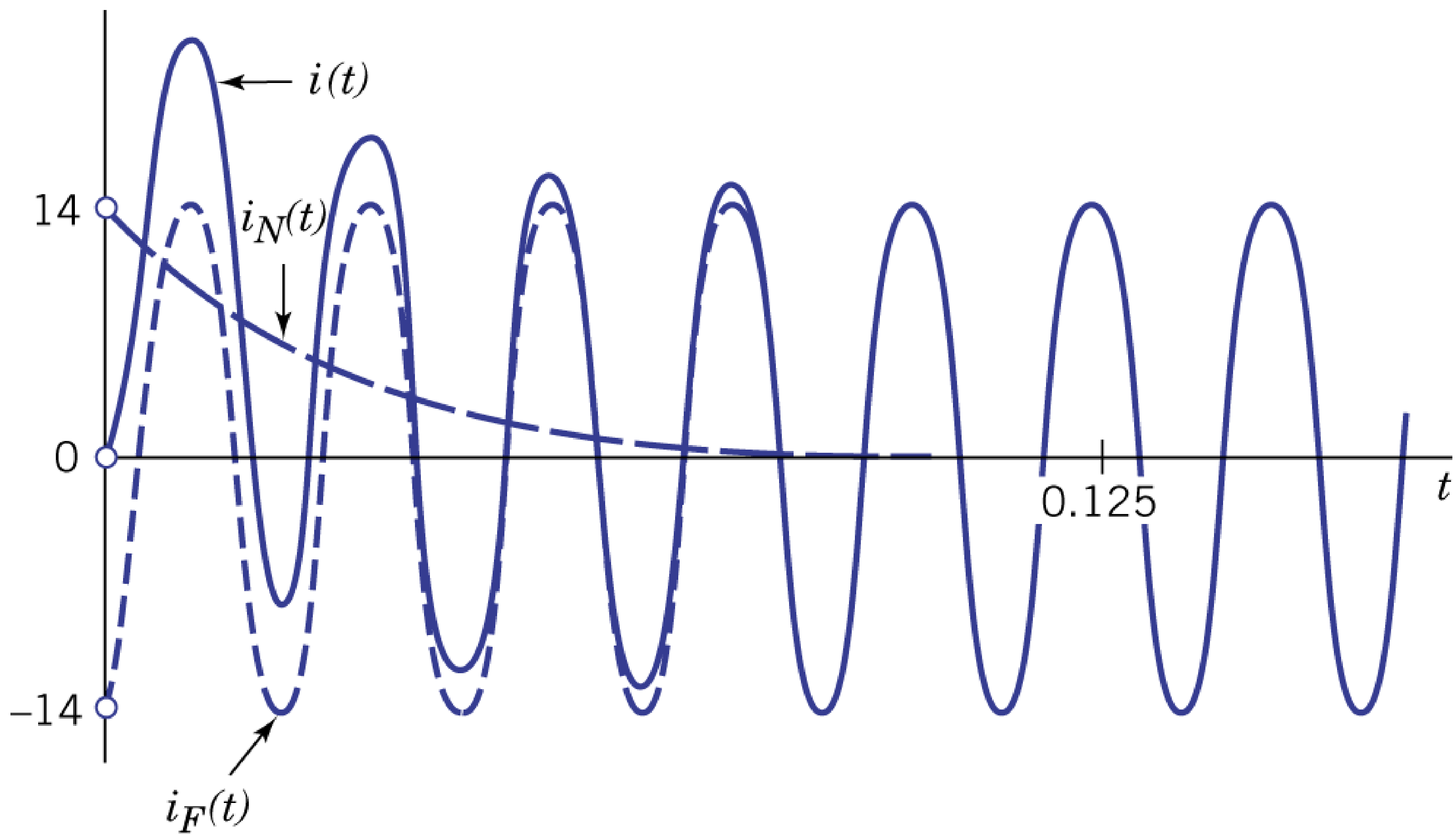
$$\text{particular solution}: i_F(t) = -14\cos 280t + 2\sin 280t$$

$$\text{complete response}: i(t) = i_N(t) + i_F(t) = Ae^{-40t} - 14\cos 280t + 2\sin 280t$$

$$i(0^+) = 0 = -14 + A$$

$$A = 14$$

Example 5.12: Complete Response Calculation (RL)



Chapter 5: Problem Set

- 42, 44, 45, 48, 54, 61, 64