Chapter 11 Frequency Response and Filters

Chapter 11: Outline

Complex Frequency → Real Frequency →

Frequency Response (amplitude and phase)

Plots of Frequency Response

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Filters (frequency selectivity)

Characteristics: Low, High, Bandpass and Notch

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Active Filters (implemented with op-amps)

↓

Bode Plots (approx. interpretation and build up)

Complex Frequency

- Complex frequency: oscillating voltages or currents with exponential amplitudes.

$$x(t) = X_m e^{st} \cos(wt + f_x)$$

$$= \mathrm{Re}\left[X_m e^{st} e^{j(wt + f_x)} \right] = \mathrm{Re}\left[(X_m e^{jf_x}) e^{(s\ +\ jw)t} \right]$$

$$\text{Complex frequency}: s \equiv s\ +\ jw$$

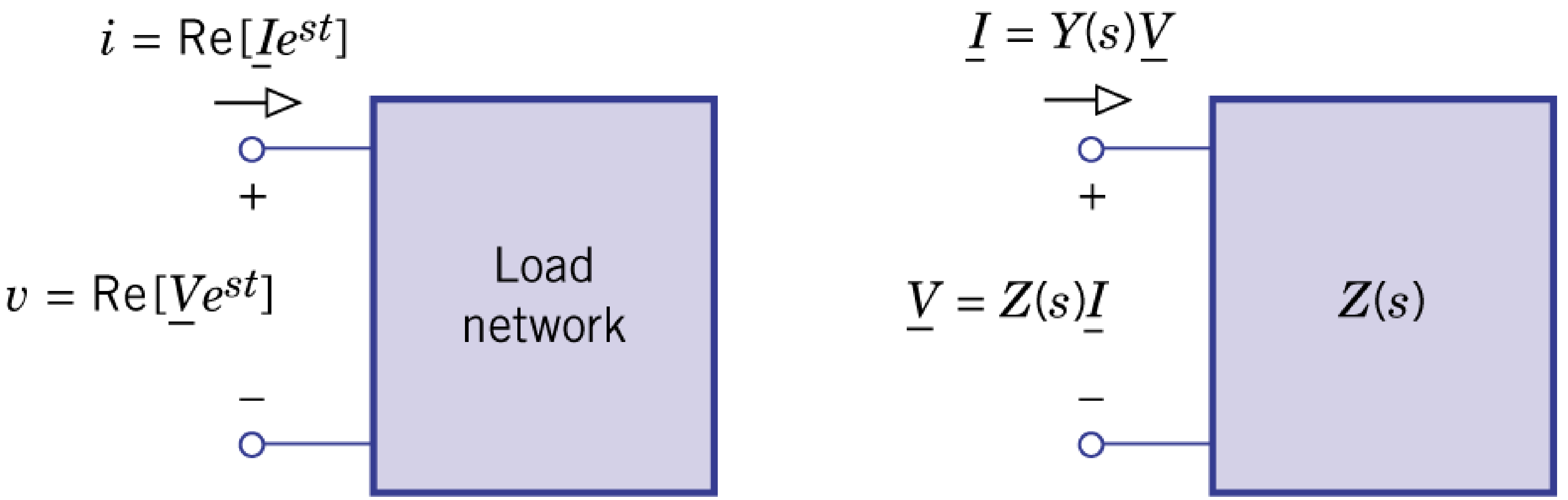
$$\text{Phasor}: \underline{X} \equiv X_m \angle f_x = X_m e^{jf_x}$$

$Z(s) \equiv \underline{V} / \underline{I}$

Generalized Impedance and Admittance

$$Z(s) \equiv \underline{V} / \underline{I}$$

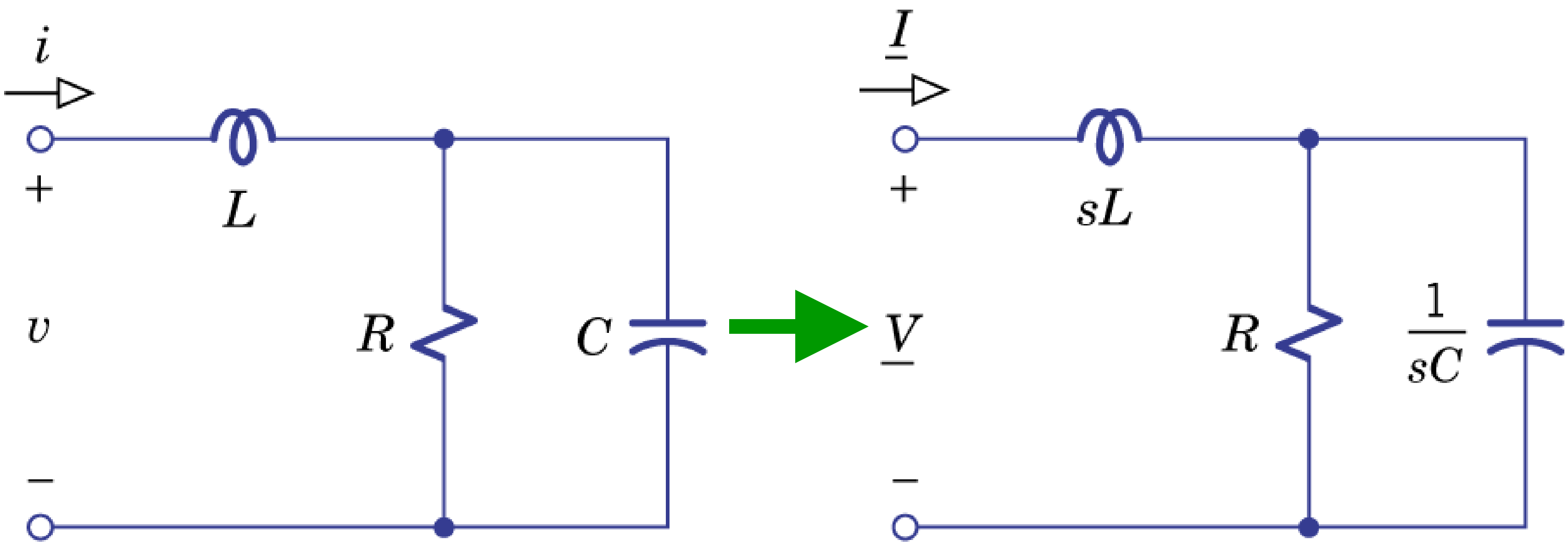
$$Y(s) \equiv 1 / Z(s) = \underline{I} / \underline{V}$$



(a) Load network *(b)* *s*-domain diagram

$Z(s) \equiv \underline{V} / \underline{I}$

Generalized Impedance and Admittance



$jw \rightarrow s$

Network Function

- Any response forced by a complex-frequency excitation.

$$\text{Input}: x(t) = X_m e^{st} \cos(wt + f_x) = \mathrm{Re}\left[\underline{X} e^{st} \right]$$

$$\left(\underline{X} \equiv X_m \angle f_x = X_m e^{jf_x} \right)$$

$$\text{Response}: y(t) = Y_m e^{st} \cos(wt + f_y) = \mathrm{Re}\left[\underline{Y} e^{st} \right]$$

$$\left(\underline{Y} \equiv Y_m \angle f_Y = Y_m e^{jf_Y} \right)$$

$$\text{Network function}: H(s) \equiv \underline{Y} / \underline{X}$$

Network Function (Rational)

$$y' = \frac{d}{dt}\mathrm{Re}\left[\underline{Y}e^{st}\right] = \mathrm{Re}\left[\underline{Y}\frac{de^{st}}{dt}\right] = \mathrm{Re}\left[s\underline{Y}e^{st}\right]$$

$$y \leftrightarrow \underline{Y} \Rightarrow y' \leftrightarrow s\underline{Y} \Rightarrow y'' \leftrightarrow s^2\underline{Y} \cdots$$

For an n - th order network,

$$a_n \frac{d^n y}{dt^n} + \cdots + a_1 \frac{dy}{dt} + a_0 y = b_m \frac{d^m x}{dt^m} + \cdots + b_1 \frac{dx}{dt} + b_0 x$$

$$\text{Network function} : H(s) \equiv \frac{\underline{Y}}{\underline{X}} = \frac{b_m s^m + b_{m-1}s^{m-1} + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + a_1 s + a_0}$$

Impedance and admittance are special cases.

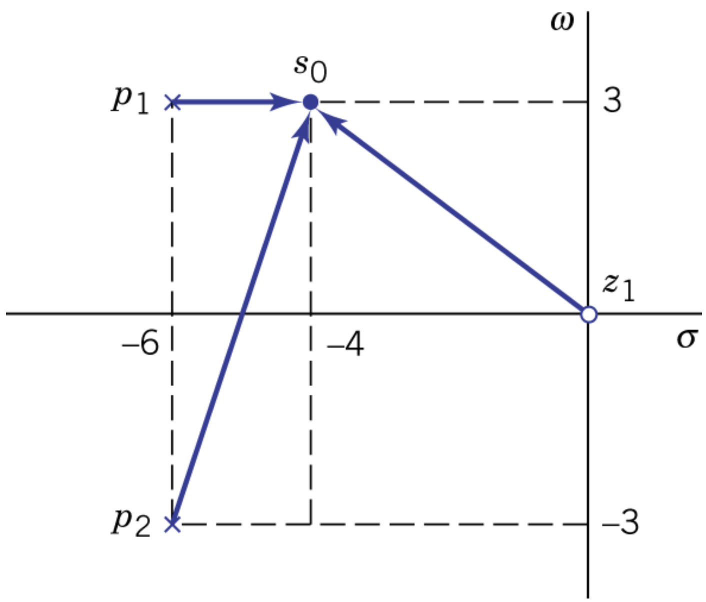
Network Function

Zero

$$\underline{Y} = H(s)\underline{X} = \frac{b_m s^m + b_{m-1}s^{m-1} + b_1 s + b_0}{a_n s^n + a_{n-1}s^{n-1} + a_1 s + a_0}\underline{X} = K\frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}\underline{X}$$

Pole

$s = s + jw$ is the input complex frequency

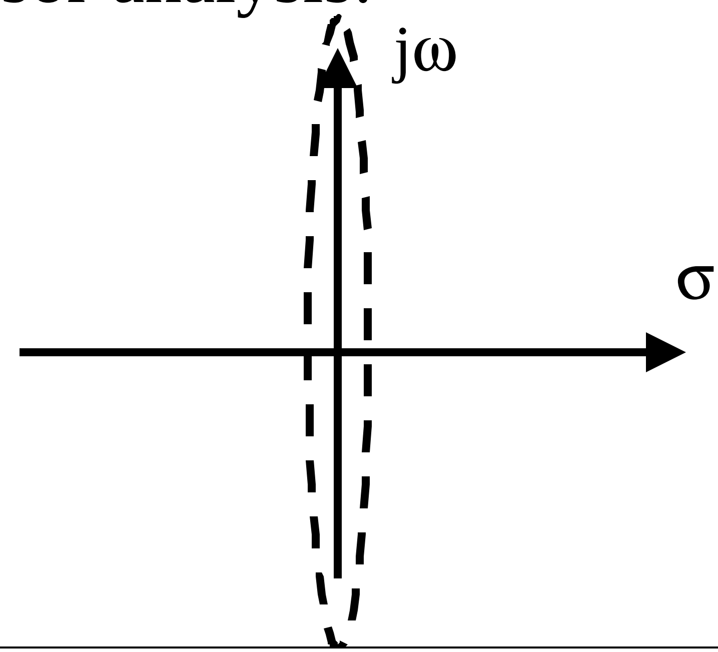


Frequency Response

Frequency Response

- Frequency response is the forced response of a circuit to a sinusoid ac waveform of a particular frequency. Amplitude ratio and phase shift are typically used to characterize frequency response.
- Transfer function vs. phasor analysis:

$$H(s) \to H(jw)$$



Frequency Response

$$x(t) = X_m \cos(wt + f_x) = \mathrm{Re}\left[\underline{X} e^{jwt}\right]$$

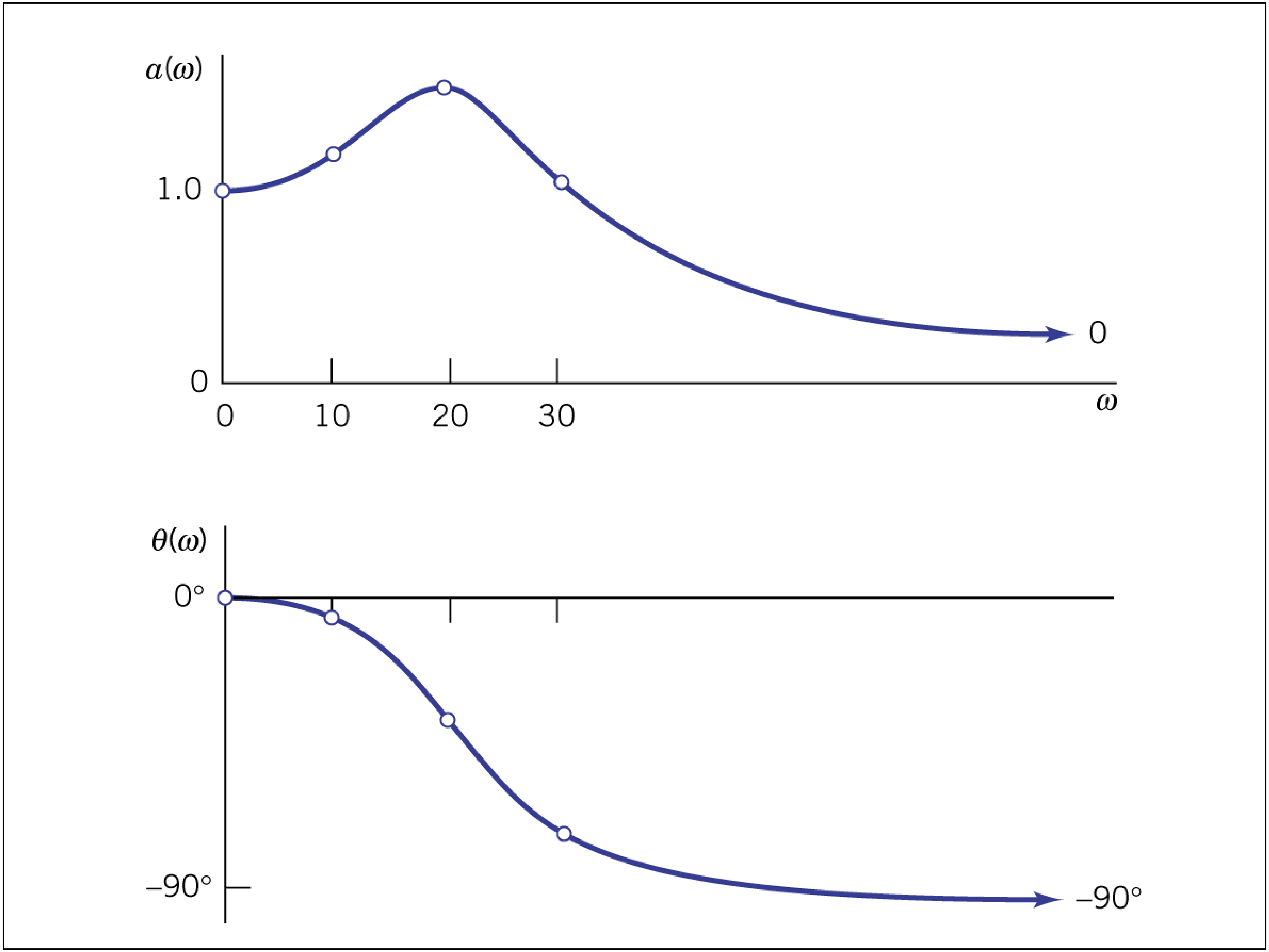
$$y(t) = Y_m \cos(wt + f_y) = \mathrm{Re}\left[\underline{Y} e^{jwt}\right]$$

$$\underline{Y} = H(jw)\underline{X}$$

$$a(w) \equiv \left|H(jw)\right| = Y_m / X_m$$:Amplitude ratio

$$q(w) \equiv \angle H(jw) = f_y - f_x$$:Phase shift

Functions of frequency



a(ω)
1.0
0
0
10
20
30
ω
0
θ(ω)
0°
–90°
–90°

Superposition

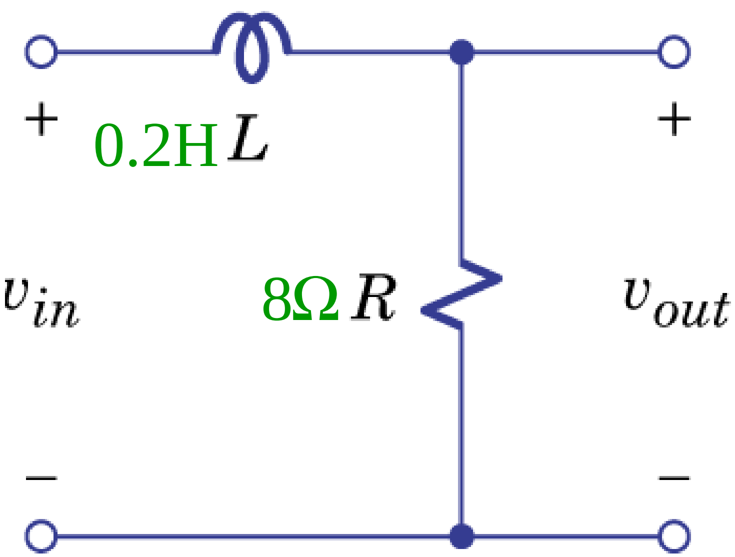
- Superposition for waveforms at different frequencies:

$$x(t) = X_1 \cos(w_1 t + f_1) + X_2 \cos(w_2 t + f_2) + \cdots$$

$$y(t) = a(w_1) X_1 \cos(w_1 t + q(w_1) + f_1) + a(w_2) X_2 \cos(w_2 t + q(w_2) + f_2) + \cdots$$

→ Phasor analysis at different frequencies

Example 11.1: A Frequency-Selective Network



$$H(s) = \frac{V_{out}}{V_{in}} = \frac{R}{sL+R} \Rightarrow H(jw) = \frac{40}{40+jw}$$

$$\left.\begin{array}{l} a(w) = |H(jw)| = \dfrac{40}{\sqrt{1600+w^2}} \\ q(w) = \angle H(jw) = -\tan^{-1}\dfrac{w}{40} \end{array}\right\} w = 20{,}300$$

$$v_{out}(t) = 8.94\cos(20t - 26.6^0) + 1.32\cos(300t - 82.4^0)$$

$$v_{in} = 10\cos 20t + 10\cos 300t$$

Frequency Response Curves

- Plots of amplitude ratio and phase shift vs. frequency. They can be obtained by analytical method or graphical method.

$$a(w) = |K| \frac{|jw - z_1||jw - z_2|\cdots}{|jw - p_1||jw - p_2|\cdots}$$

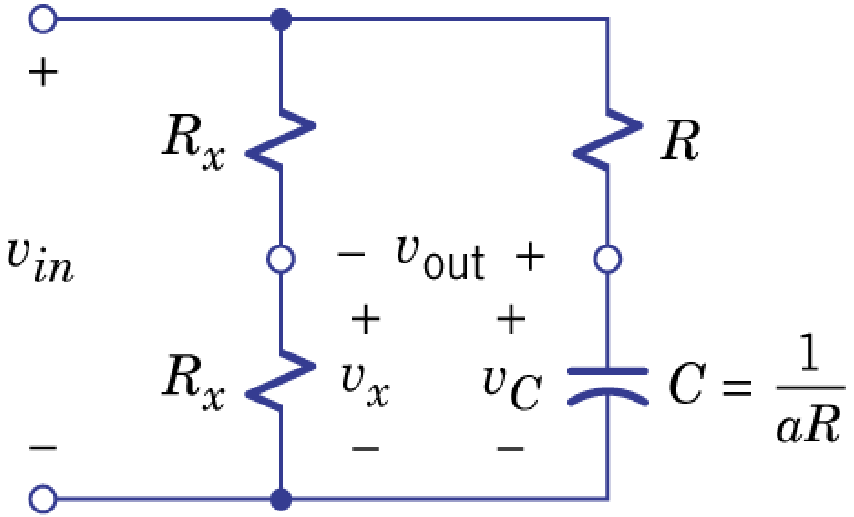
$$q(w) = \angle K + [\angle(jw - z_1) + \angle(jw - z_2) + \cdots] - [\angle(jw - p_1) + \angle(jw - p_2) + \cdots]$$

At very high frequency ($w \to \infty$):

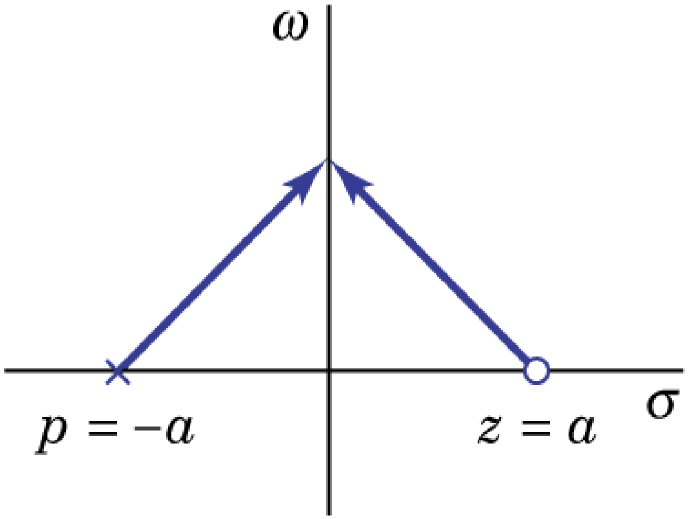
$$a(w) = \begin{cases} |K|, m = n \\ 0, m < n \end{cases}$$

$$q(w) = \angle K + (m - n) \times 90^0$$

Example 11.2: An All-Pass Network



(a) All–pass network

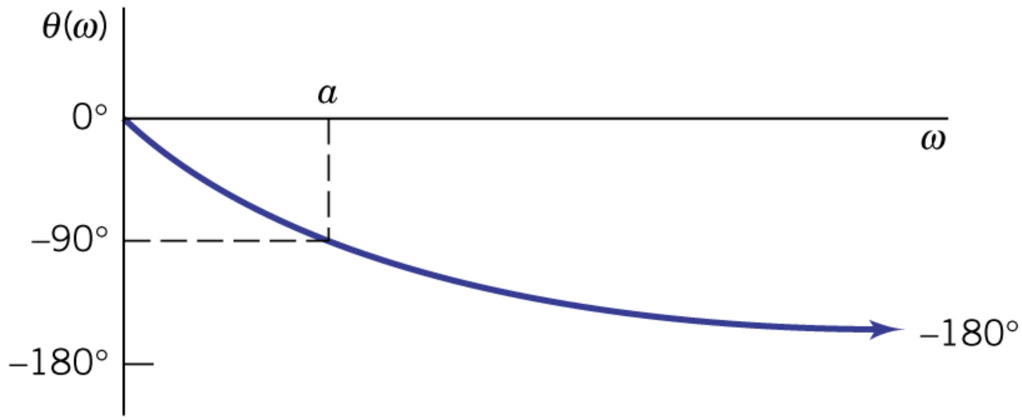


(b) Pole–zero pattern

$$H(s) = \frac{\underline{V}_{out}}{\underline{V}_{in}} = -\frac{1}{2}\frac{s-a}{s+a}\left(\equiv K\frac{s-z}{s-p}\right), \text{ where } a = \frac{1}{RC}$$

$$H(jw) = -\frac{1}{2}\frac{jw-a}{jw+a}$$

Example 11.2: An All-Pass Network



(c) Phase shift versus **ω**

$$a(w) = \left| H(jw) \right| = \frac{1}{2} \quad \text{(all pass)}$$

$$q(w) = \angle H(jw) = \tan^{-1}\left(-\frac{w}{a}\right) - \tan^{-1}\left(\frac{w}{a}\right) = -2\tan^{-1}\left(\frac{w}{a}\right)$$

Q: What does non-linear phase do?

A: Waveform Distortion

Non-Linear Phase

$$x(t) = X_1 \cos(w_1 t) + X_2 \cos(w_2 t)$$

$$y(t) = X_1 \cos(w_1 t + q(w_1)) + X_2 \cos(w_2 t + q(w_2))$$

$$= X_1 \cos(w_1(t + \frac{q(w_1)}{w_1}) + X_2 \cos(w_2(t + \frac{q(w_2)}{w_2})$$

$$= X_1 \cos(w_1(t + t\ ) + X_2 \cos(w_2(t + t\ ), \text{only if } q(w) = kw$$

Example 11.3: Frequency-Response Calculations (Analytic Method)

$$H(s)=\frac{20(s+25)}{s^2+20s+500}$$

$$H(jw)=\frac{20(25+jw)}{(500-w^2)+j20w}$$

$$a(w)=\frac{20\sqrt{625+w^2}}{\sqrt{\left(500-w^2\right)^2+400w^2}}$$

$$q(w)=\tan^{-1}\frac{w}{25}-\tan^{-1}\frac{20w}{500-w^2},w^2<500$$

$$=\tan^{-1}\frac{w}{25}\pm 180^0+\tan^{-1}\frac{20w}{500-w^2},w^2>500$$

Example 11.3 (Graphical Method)

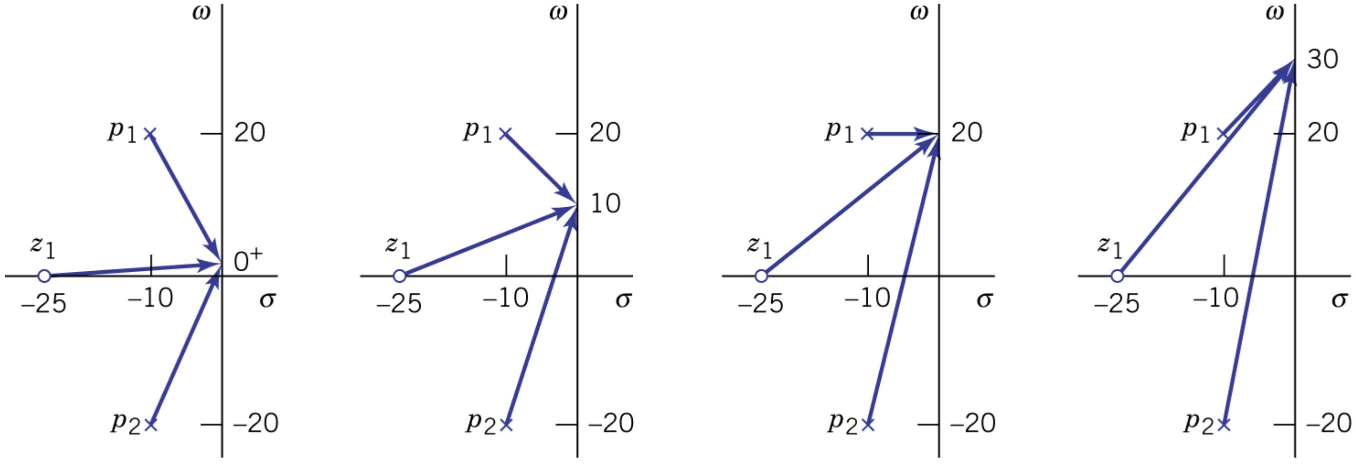
$$K = 20$$

$$z_1 = -25$$

$$p_1, p_2 = -10 \pm j20$$

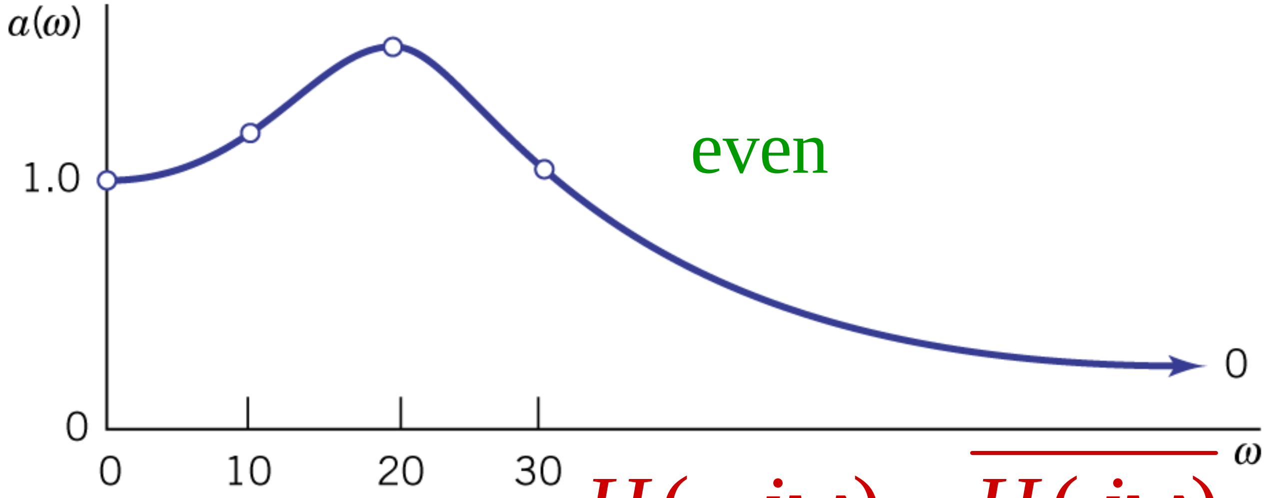
$$a(w) = 20 \frac{|jw - z_1|}{|jw - p_1||jw - p_2|}$$

$$q(w) = \angle(jw - z_1) - \angle(jw - p_1) - \angle(jw - p_2)$$



(a) (b) (c) (d)

Example 11.3 (Cont.)

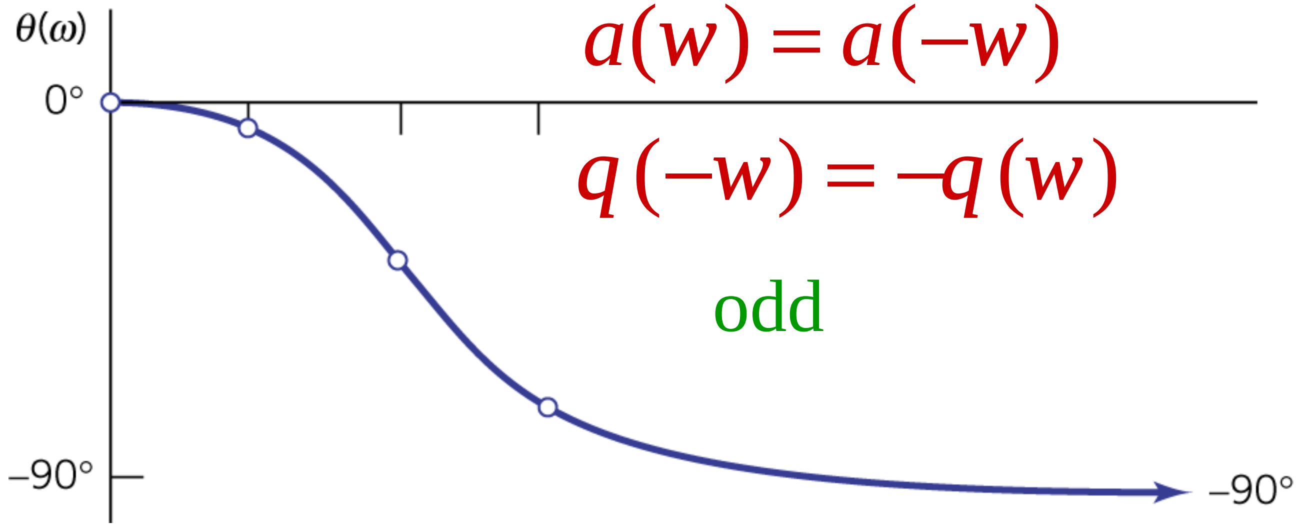


even

$$H(-jw) = \overline{H(jw)}$$

$$a(w) = a(-w)$$

$$q(-w) = -q(w)$$



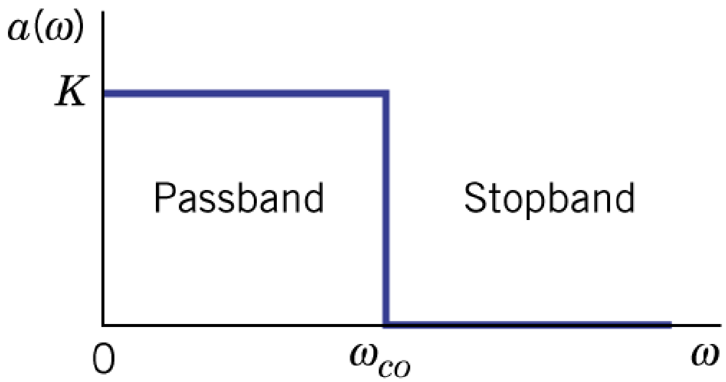
odd

Table 11.2

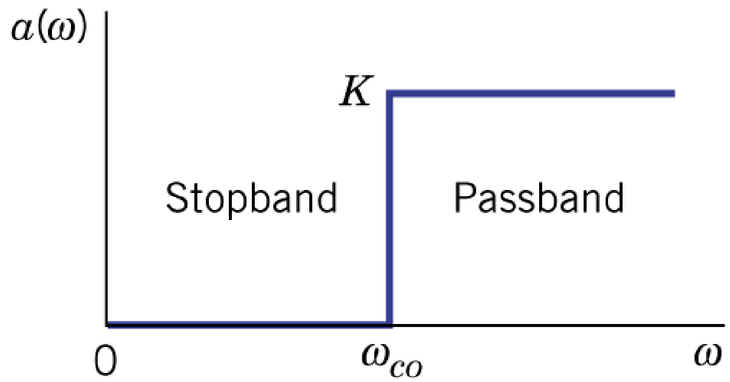
Filters

Filters

- Filters are frequency-selective networks that pass certain frequencies but suppress/reject the others.
- Four common categories: lowpass, highpass, bandpass and notch.
- A positive gain constant *K* is assumed.
- Ideal lowpass filter, ideal highpass filter, cutoff frequency, passband and stop band.



(a) Ideal lowpass filter



(b) Ideal highpass filter

First-Order Lowpass Filter

$$H_{lp}(s) = \frac{Kw_{co}}{s + w_{co}}$$

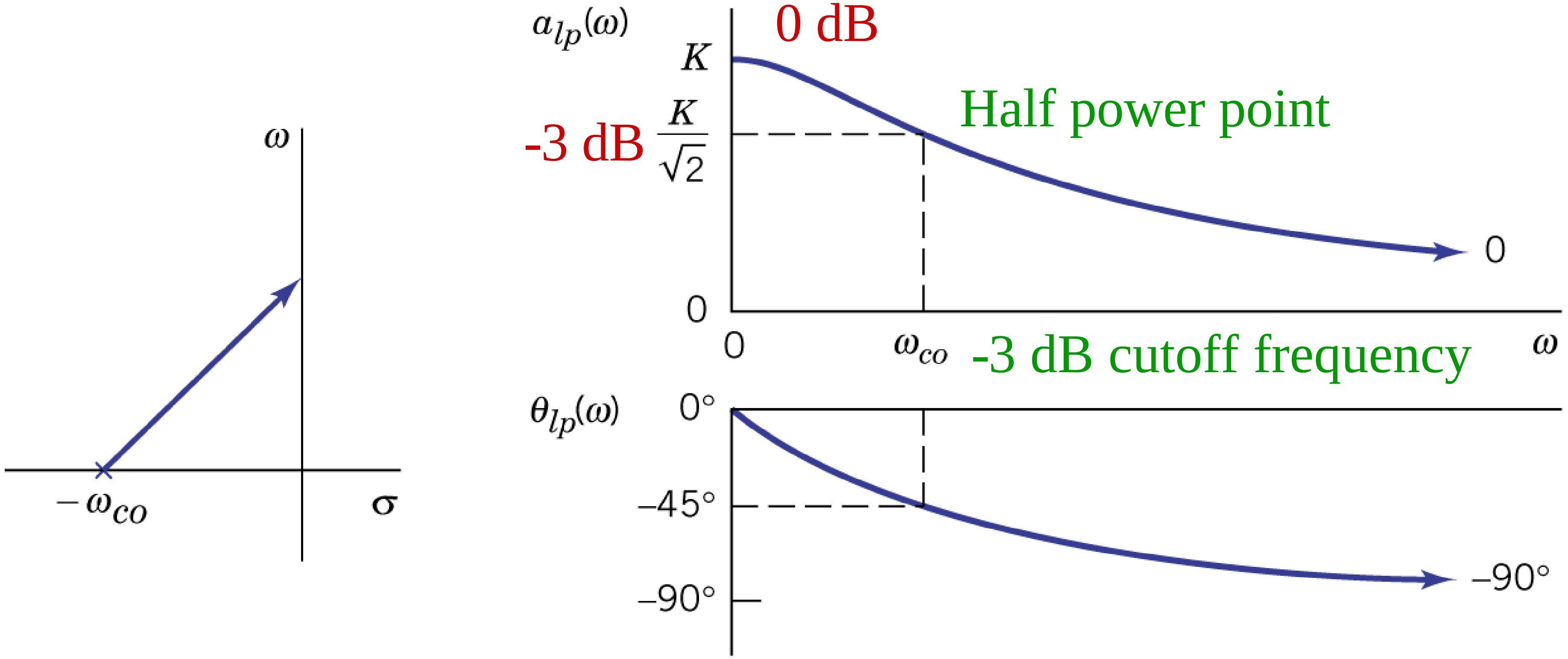
$$\Rightarrow H_{lp}(jw) = \frac{Kw_{co}}{jw + w_{co}} = \frac{K}{1 + j(w / w_{co})}$$

$$a_{lp}(w) = \frac{K}{\sqrt{1 + (w / w_{co})^2}}$$

$$q_{lp}(w) = -\tan^{-1}\frac{w}{w_{co}}$$

K is positive (low frequency gain)

First-Order Lowpass Filter



(a) s–plane diagram *(b)* Frequency response curves

First-Order Highpass Filter

$$H_{hp}(s) = \frac{Ks}{s + w_{co}}$$

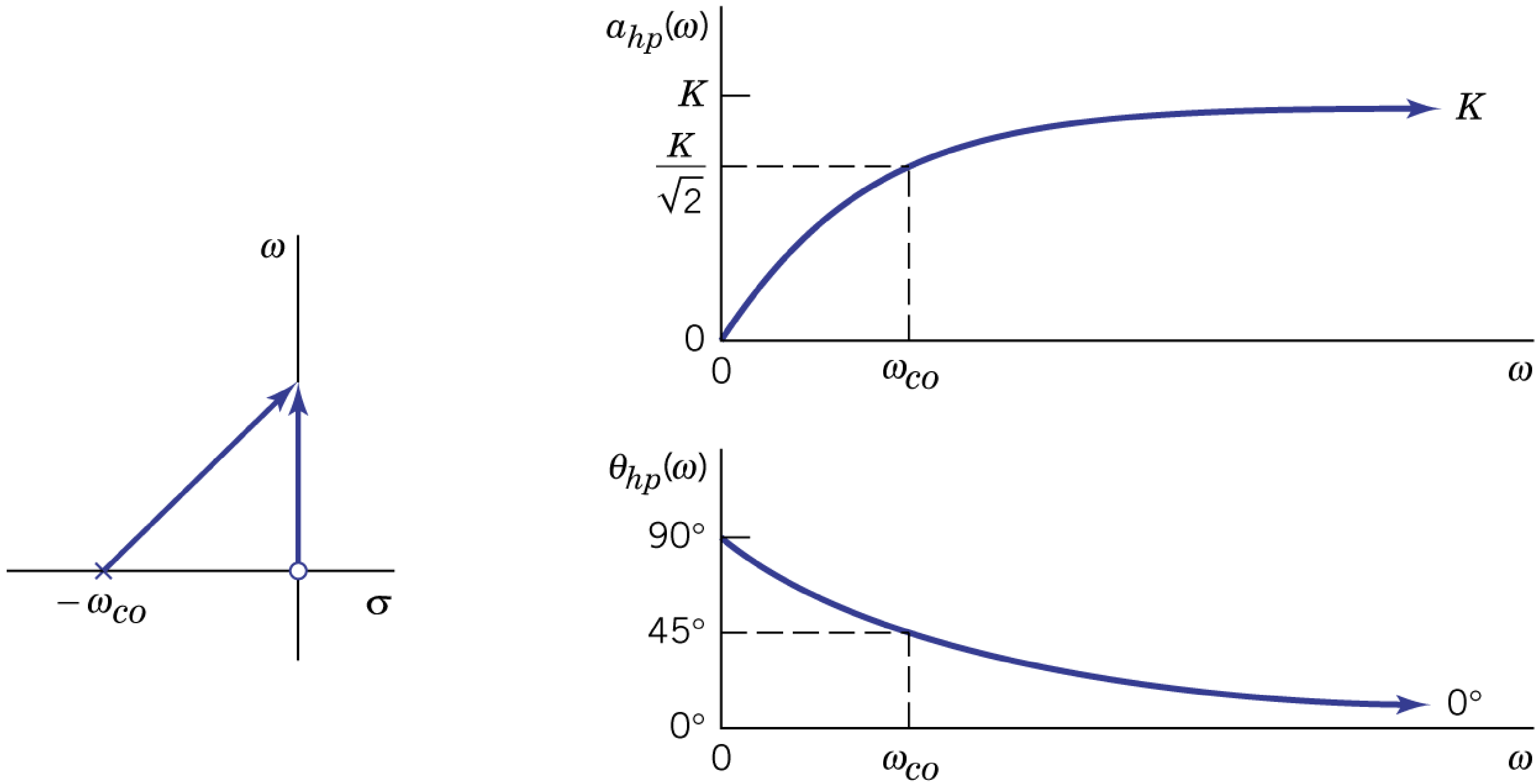
$$\Rightarrow H_{hp}(jw) = \frac{Kjw}{jw + w_{co}} = \frac{K}{1 - j(w_{co}/w)}$$

$$a_{hp}(w) = \frac{K}{\sqrt{1 + (w_{co}/w)^2}}$$

$$q_{hp}(w) = -\tan^{-1}\frac{w_{co}}{w}$$

K : high frequency gain

First-Order Highpass Filter



(a) s–plane diagram

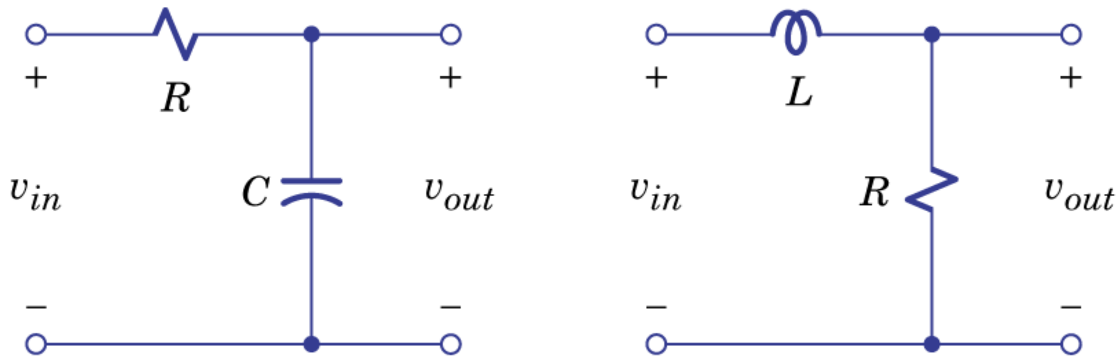
(b) Frequency response curves

radian frequency w vs. cyclical frequency f

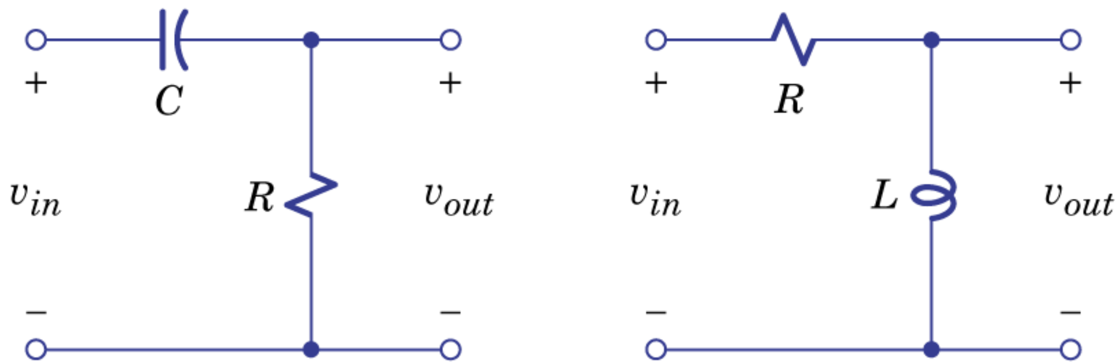
$$w=2pf$$

$$w/\,w_{co}=f/f_{co}$$

First-Order Filter Networks



(a) Lowpass filters



(b) Highpass filters

$K = 1$

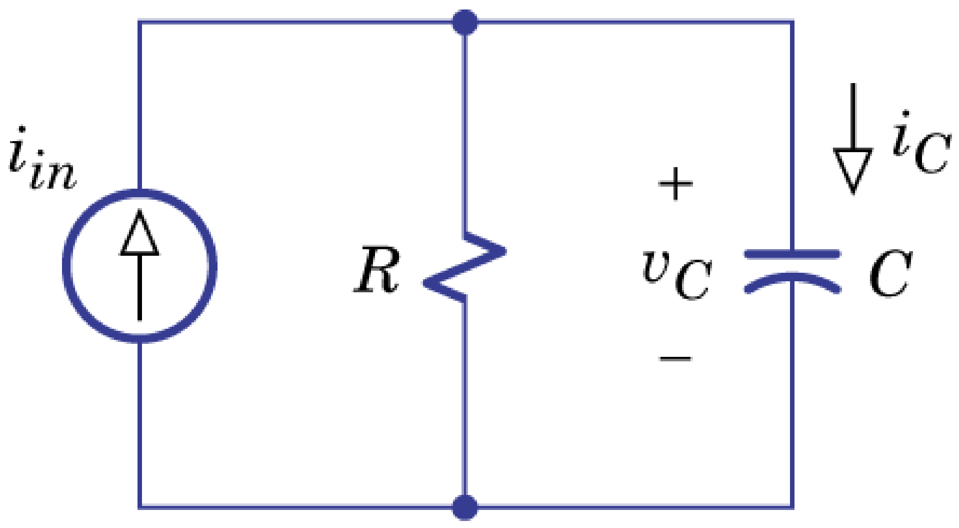
$$w_{co} = \frac{1}{t}, \quad t = \begin{cases} RC \\ L/R \end{cases}$$

For example:

RC lowpass filter

$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{\frac{1}{RC}}{s + \frac{1}{RC}}$$

Example 11.4: Parallel Filter Network

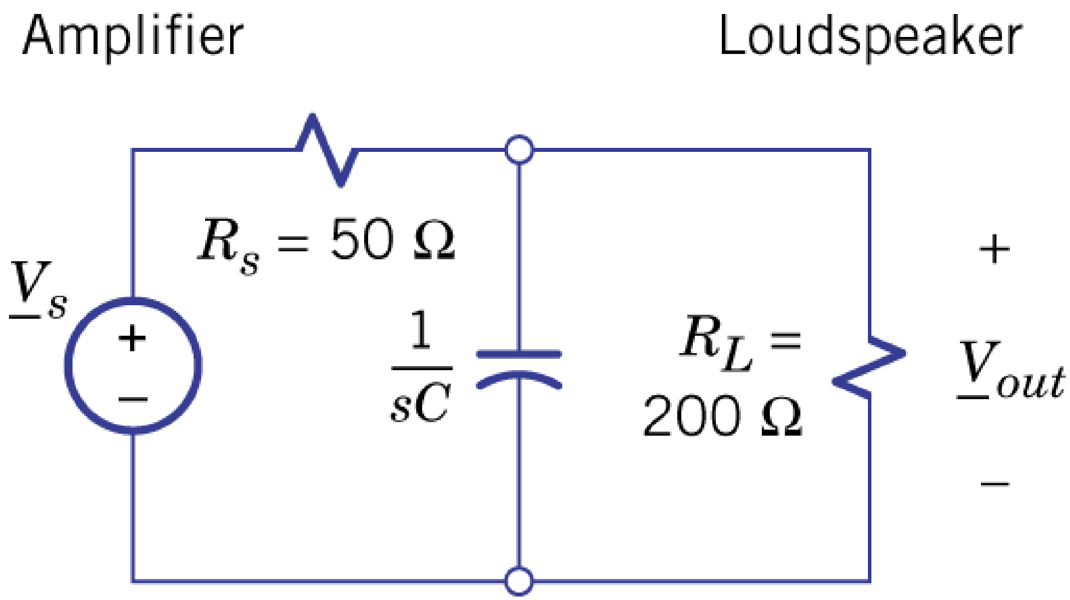


$$H(s) = \frac{\underline{I}_c}{\underline{I}_{in}} = \frac{s}{s + \frac{1}{RC}}$$

$$H(jw) = \frac{jw}{jw + \frac{1}{RC}} = \frac{1}{1 - j\frac{1}{RCw}}$$

$$K = 1, w_{co} = \frac{1}{RC}$$

Example 11.5: Design of a Lowpass Filter



Design a low pass filter with f_{co} around 4 KHz

$$H(s) = \frac{\underline{V}_{out}}{\underline{V}_s} = \frac{G_s}{sC + G_s + G_L} = \frac{G_s / C}{s + (G_s + G_L)/C} = \frac{Kw_{co}}{s + w_{co}}$$

Example 11.5: (Cont.)

$$H(s) = \frac{\underline{V}_{out}}{\underline{V}_s} = \frac{G_s}{sC + G_s + G_L} = \frac{G_s / C}{s + (G_s + G_L)/C} = \frac{Kw_{co}}{s + w_{co}}$$

$$w_{co} = \frac{G_s + G_L}{C} = \frac{1}{40C} = 2pf_{co} = 2p \cdot 4KHz$$

$$K = \frac{G_s}{G_s + G_L} = 0.8 \ \text{(loading effect)}$$

$$C = \frac{1}{40} w_{co} \approx 1mF$$

$$w_{co} = \frac{1}{t}, \text{ where } t = R_{eq}C, R_{eq} = R_s \| R_L$$

Example 11.5: (Cont.)

Let $v_s(t) = 5\cos w_1 t + 0.5\cos w_2 t, \quad w_1 = 2p \cdot 3k, w_2 = 2p \cdot 16k$

$H(jw_1) = 0.64\angle -37^0, H(jw_2) = 0.19\angle -76^0$

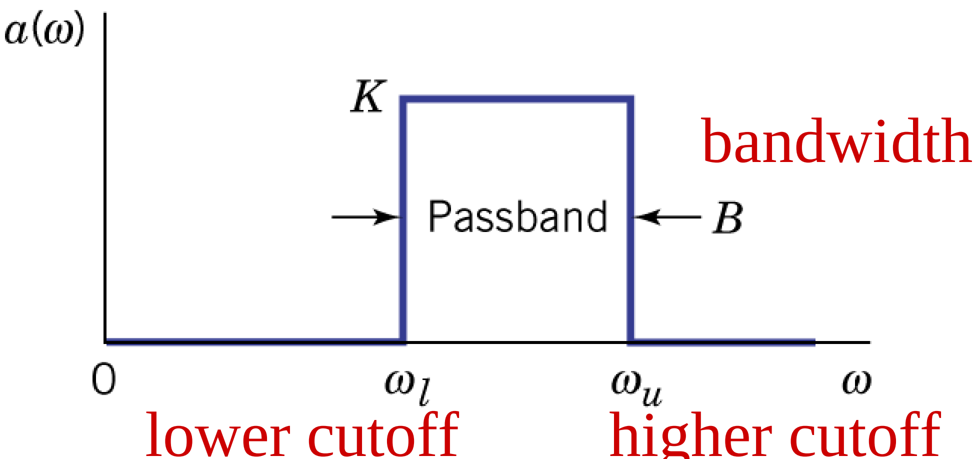
$v_{out}(t) = 3.2\cos(w_1 t - 37^0) + 0.095\cos(w_2 t - 76^0)$

$(10\% \rightarrow 3\%)$

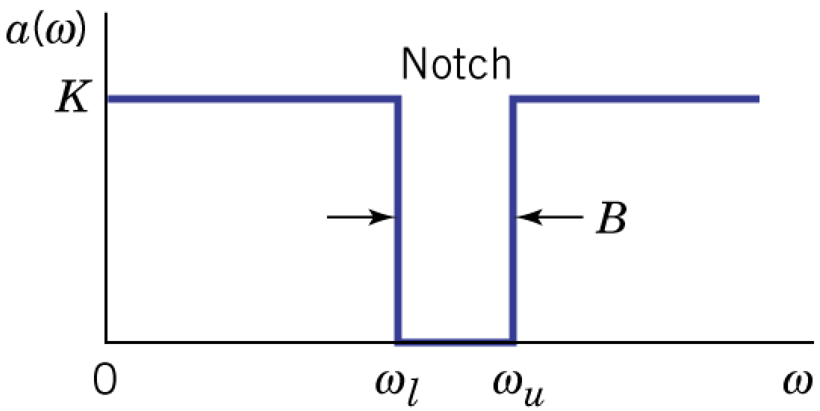
Bandpass and Notch Filters

Bandpass and Notch Filters

- Ideal bandpass filter, ideal notch filter (band-reject filter), lower cutoff frequency, upper cutoff frequency and bandwidth.



(a) Ideal bandpass filter



(b) Ideal notch filter

Quality Factor

- Second order bandpass filter and quality factor.

$$H_{bp}(s) = \frac{K(w_0 / Q)s}{s^2 + (w_0 / Q)s + w_0^2}$$

$$Q = w_0 / 2a \quad (a : \text{damping coefficient})$$

$$\text{when underdamped} : a < w_0 \text{ (i.e., } Q > 1/2)$$

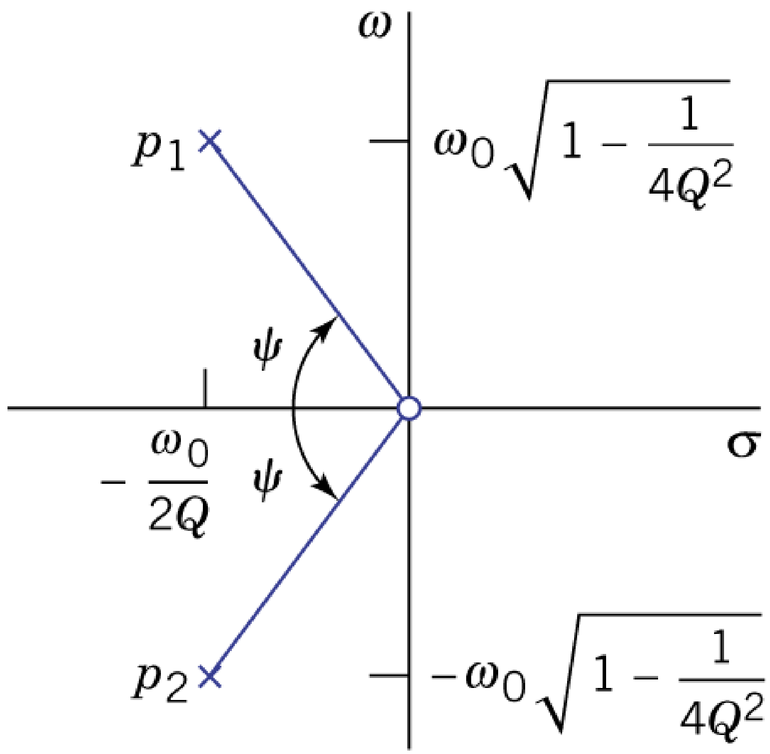
$$p_1, p_2 = -\frac{w_0}{2Q} \pm j w_0 \sqrt{1 - \frac{1}{4Q^2}}$$

$$|p_1| = |p_2| = w_0$$

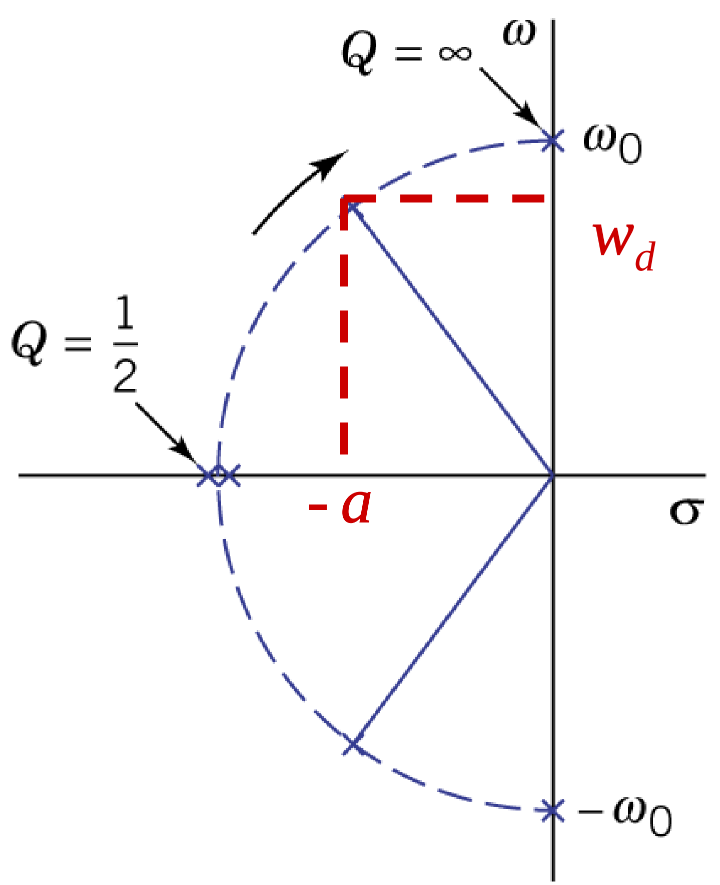
$$\angle p_1 = 180^0 - y \;, \angle p_2 = 180^0 + y$$

$$y = \cos^{-1}(1/2Q)$$

Quality Factor

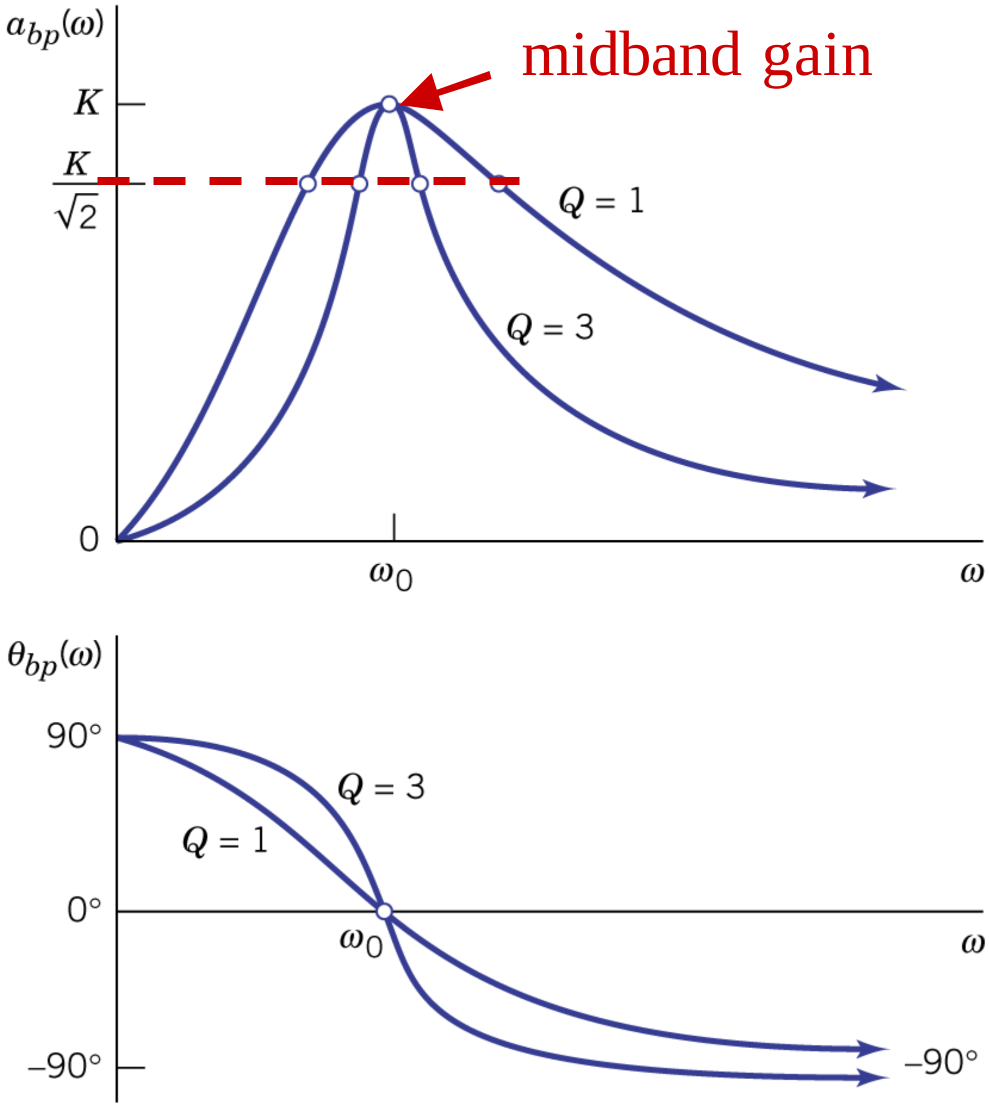


(a) Pole–zero pattern for second–order bandpass filter



(b) Pole movement as Q changes

Quality Factor



$$H_{bp}(jw) = \frac{K}{1 + jQ\left(\frac{w}{w_0} - \frac{w_0}{w}\right)}$$

$$a_{bp}(w) = \frac{K}{\sqrt{1 + Q^2\left(\frac{w}{w_0} - \frac{w_0}{w}\right)^2}}$$

$$q_{bp}(w) = -\tan^{-1} Q\left(\frac{w}{w_0} - \frac{w_0}{w}\right)$$

(-3 dB) Bandwidth

w_u and w_l at $Q\left(\frac{w}{w_0}-\frac{w_0}{w}\right)=\pm 1$

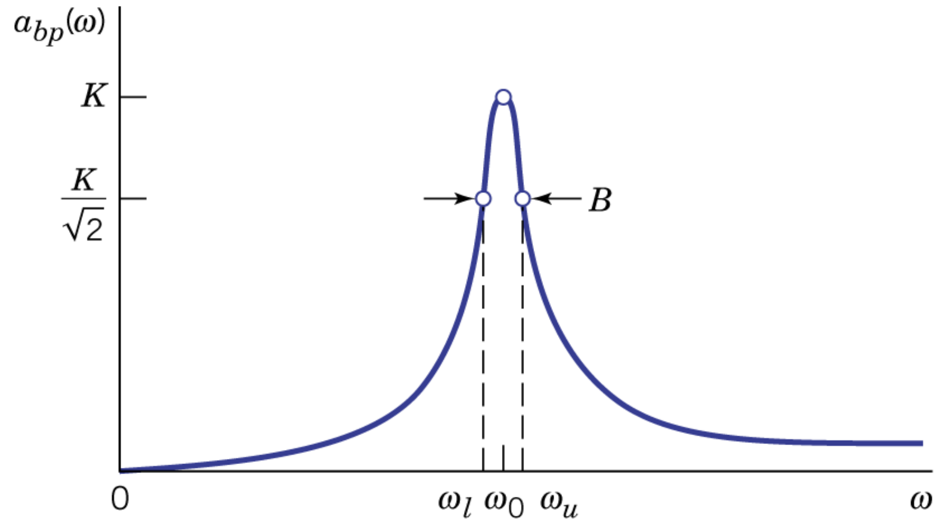
$a_{bp}(w_l)=a_{bp}(w_u)=K/\sqrt{2}$

$Q>1/2$ (for bandpass filtering)

$$w_u, w_l = w_0\sqrt{1+\frac{1}{4Q^2}} \pm \frac{w_0}{2Q}$$

$$B = w_u - w_l = \frac{w_0}{Q}$$

$w_u \cdot w_l = w_0^2$ (geometric mean)



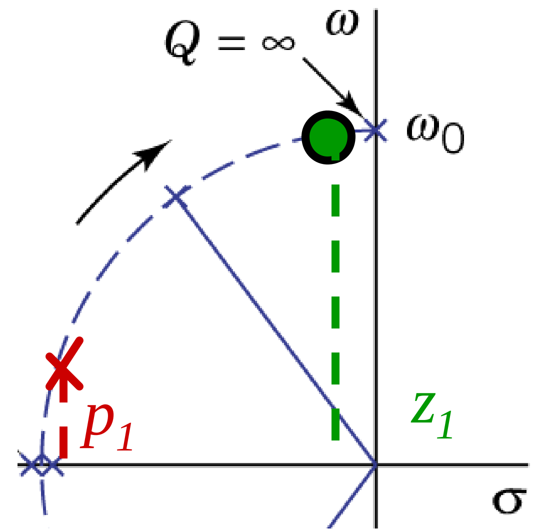
$B << w_0 \Rightarrow$ narrowband

high $Q: Q=\frac{w_0}{B}\geq 10$

$$w_u, w_l \approx w_0 \pm \frac{w_0}{2Q} = w_0 \pm \frac{1}{2}B$$

(approximately symmetric)

Second-Order Notch Filter



$$H_{no}(s) = \frac{K(s^2 + 2bs + w_0^2)}{s^2 + \frac{w_0}{Q}s + w_0^2}, \quad b << \frac{w_0}{2Q}$$

$$z_1, z_2 = -b \pm j\sqrt{w_0^2 - b^2} \approx -b \pm jw_0$$

$$-3\text{dB bandwidth}: \frac{K(w_0^2 - w^2)}{(w_0^2 - w^2) + j\frac{ww_0}{Q}} = \frac{K}{1 + j\frac{ww_0}{Q}\frac{1}{w_0^2 - w^2}} = \frac{K}{1 \pm j \cdot 1}$$

$$B = \frac{w_0}{Q}$$

Second-Order Notch Filter

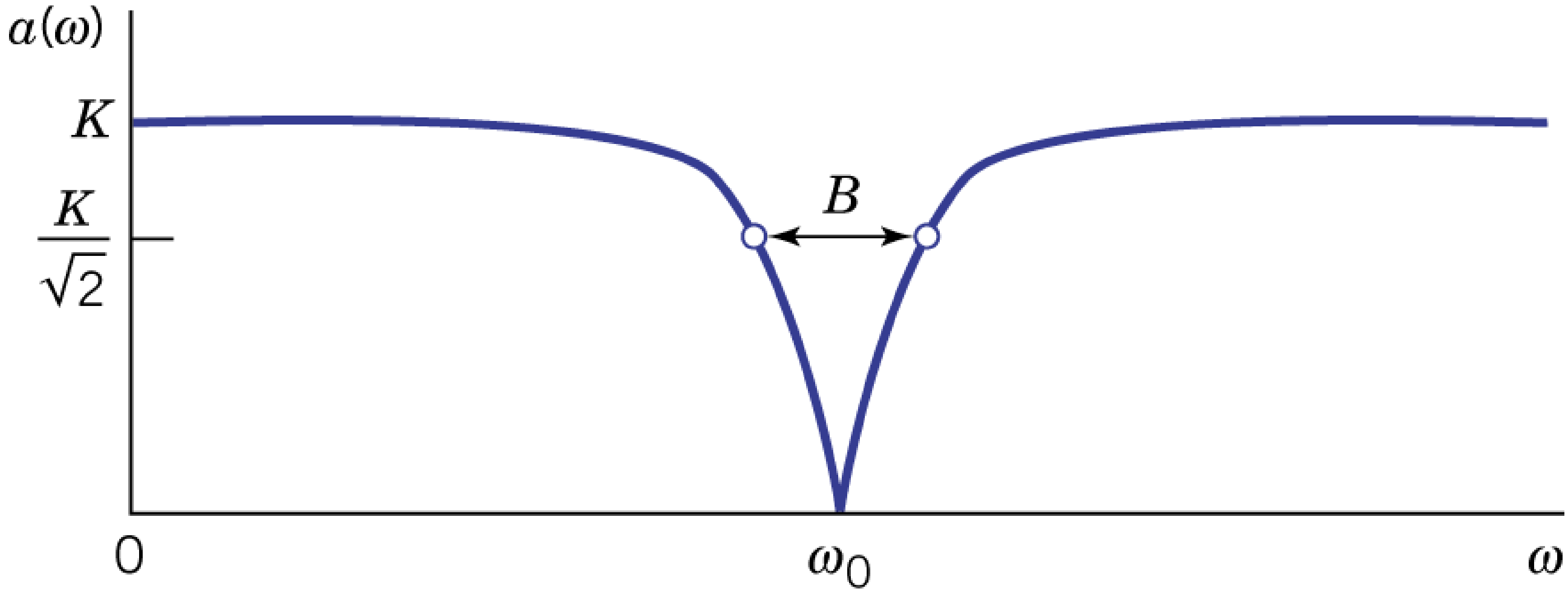


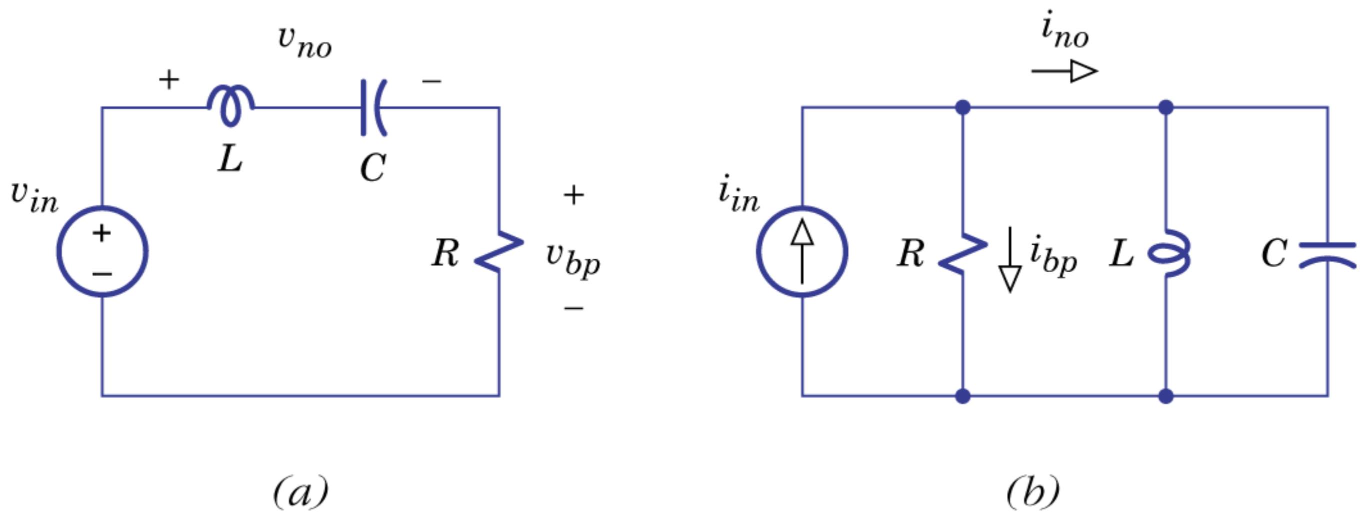
Table 11.3

TABLE 11.3 Simple Filters

Type	Transfer Function	Properties
Lowpass	$H(s) = \dfrac{K\omega_{co}}{s + \omega_{co}}$	$a(0) = K$ $a(\omega_{co}) = K/\sqrt{2}$
Highpass	$H(s) = \dfrac{Ks}{s + \omega_{co}}$	$a(\infty) = K$ $a(\omega_{co}) = K/\sqrt{2}$
Bandpass	$H(s) = \dfrac{K(\omega_0/Q)s}{s^2 + (\omega_0/Q)s + \omega_0^2}$	$a(\omega_0) = K$ $B = \omega_0/Q$
Notch	$H(s) = \dfrac{K(s^2 + 2\beta s + \omega_0^2)}{s^2 + (\omega_0/Q)s + \omega_0^2}$	$a(\omega_0) = KQ\beta/\omega_0$ $B = \omega_0/Q$

Resonant Circuits

- Resonant circuits for bandpass and notch filters.

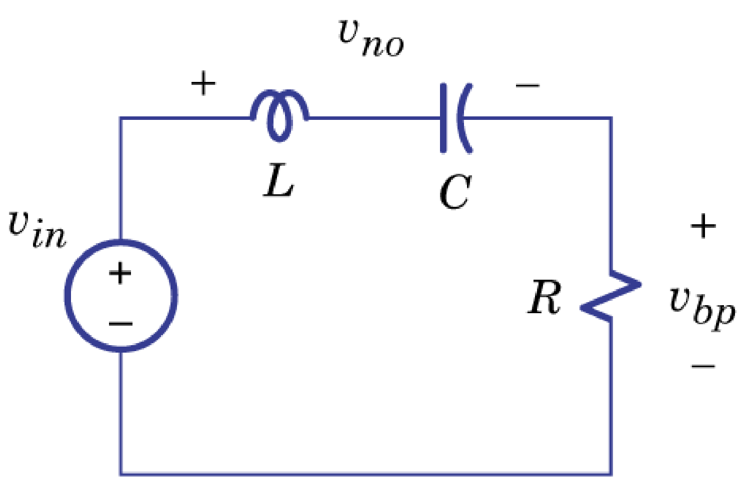


$$Q = \frac{w_0}{2a}$$

$$\text{For a series RLC network}: Q_{ser} \equiv \frac{w_0 L}{R} = \frac{1}{w_0 CR} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\text{For a parallel RLC network}: Q_{par} \equiv w_0 CR = \frac{R}{w_0 L} = R\sqrt{\frac{C}{L}}$$

Resonant Circuits



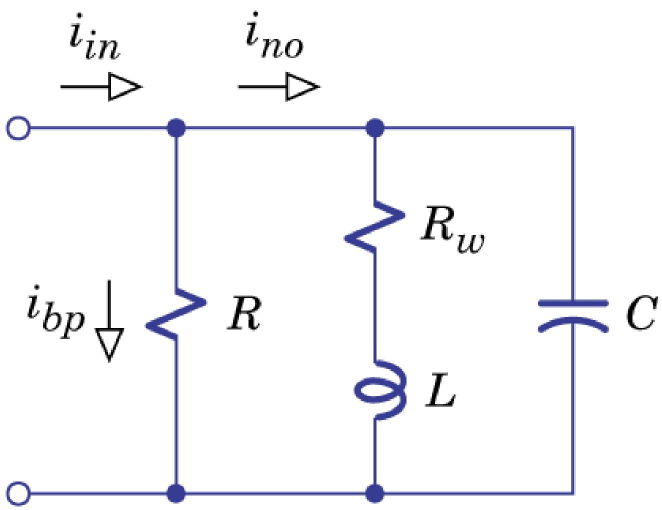
(a)

$$\frac{V_{bp}}{V_{in}} = \frac{\frac{R}{L}s}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

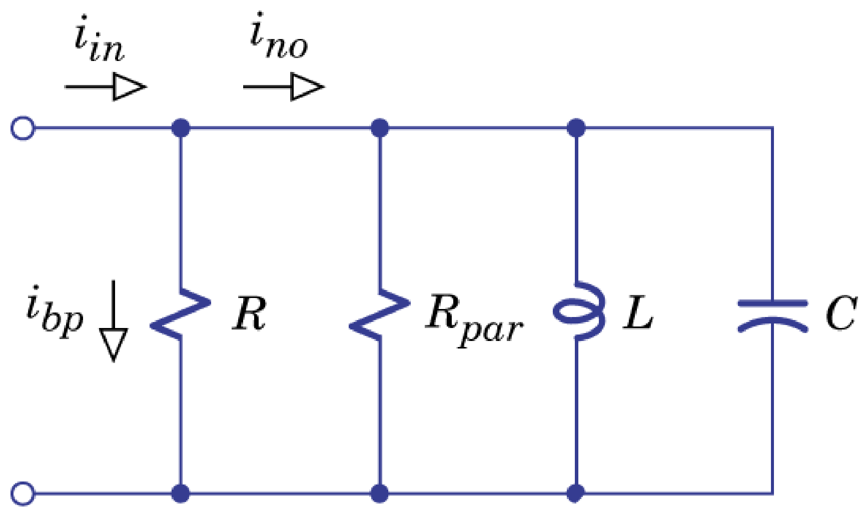
$$\frac{V_{no}}{V_{in}} = \frac{s^2 + \frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$b = 0, K = 1, w_0 = \frac{1}{\sqrt{LC}}, Q = \begin{cases} Q_{ser} \\ Q_{par} \end{cases}$$

Winding Resistance (refer to 6.4)



(a) Parallel network with winding resistance



(b) Equivalent network for $\omega \approx \omega_0$

$$\text{if } R_w << w_0 L, \quad R_{par} = L / C R_w$$

$$Q_{par} = w_0 C \left(R \| R_{par} \right) / w_0 L$$

$$Q_{par} \downarrow, \quad \text{notch width} \uparrow$$

Example 11.6: Design of a Bandpass Filter (Parallel)

$$\text{Require bandpass}: 20kHz \pm 250Hz$$

$$\text{Given } L = 1mH, R_w = 1.2\Omega, \text{ find } C \text{ and } R$$

$$Q = \frac{w_0}{B} = \frac{20k}{500} = 40 \ \ (w_0 \rightarrow \text{center frequency})$$

$$Q_{par} = Q = 40$$

$$C = \frac{1}{w_0^2 L} = 63.3nF$$

$$R \| R_{par} = Q w_0 L = 5.03k\Omega$$

$$R_{par} = L / C R_w = 13.2k\Omega$$

$$R = 8.13k\Omega$$

Bode Plots

Bode Plots

- Amplitude ratio and frequency are converted to a logarithmic scale.
- Factored functions and decibels:

$$H(s) = KH_1(s)H_2(s)\cdots$$

$$a(w) = \left|H(jw)\right| = \left|K\right| a_1(w)a_2(w)\cdots$$

$$g(w) \equiv 20\log a(w) = 20\log\left|K\right| + 20\log a_1(w) + 20\log a_2(w) + \cdots$$

$$= K_{dB} + g_1(w) + g_2(w) + \cdots$$

dB gain

0^0 or $\pm 180^0$

$$q(w) = \angle H(jw) = \angle K + q_1(w) + q_2(w) + \cdots$$

Amplitude vs. dB Gain

Amplitude ratio	10	2	$2^{1/2}$	1	$2^{-1/2}$	1/2	1/10
Gain in dB	20	6	3	0	-3	-6	-20

First-Order Factors:

Ramp Function

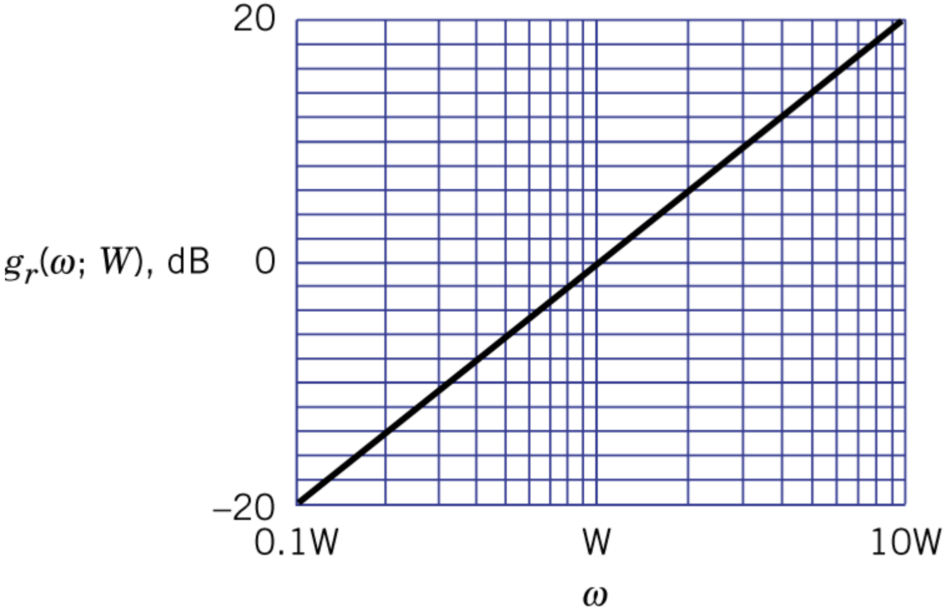
Highpass Function

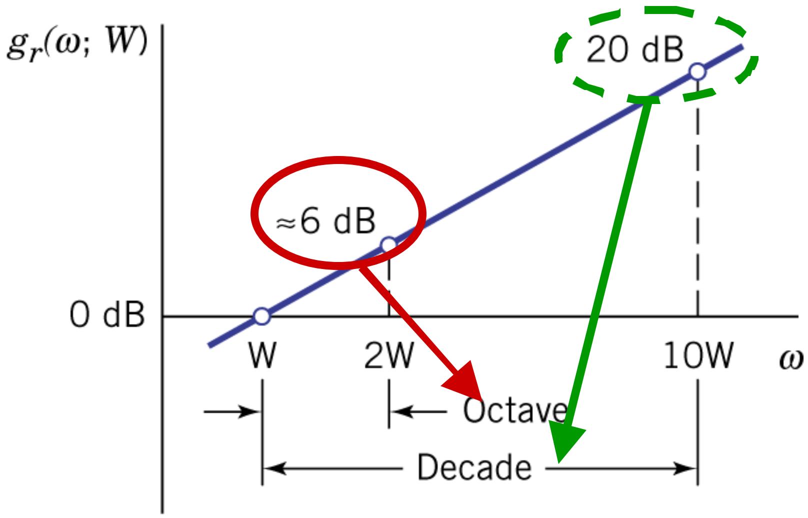
Lowpass Function

Ramp Function

$$H_r(s;W) \equiv \frac{s}{W} \Rightarrow H_r(jw;W) = \frac{jw}{W} = \frac{w}{W}\angle 90^0$$

$$g_r(w;W) = 20\log\frac{w}{W}, \quad q_r(w,W) = 90^0$$





Highpass Function

$$H_{hp}(s;W) \equiv \frac{s}{s+W} \Rightarrow H_{hp}(jw;W) \equiv \frac{j(w/W)}{1+j(w/W)}$$

$$if \quad \frac{w}{W} << 1,\ 1+j(w/W) \approx 1$$

$$\Rightarrow H_{hp}(jw;W) \approx j\frac{w}{W}$$

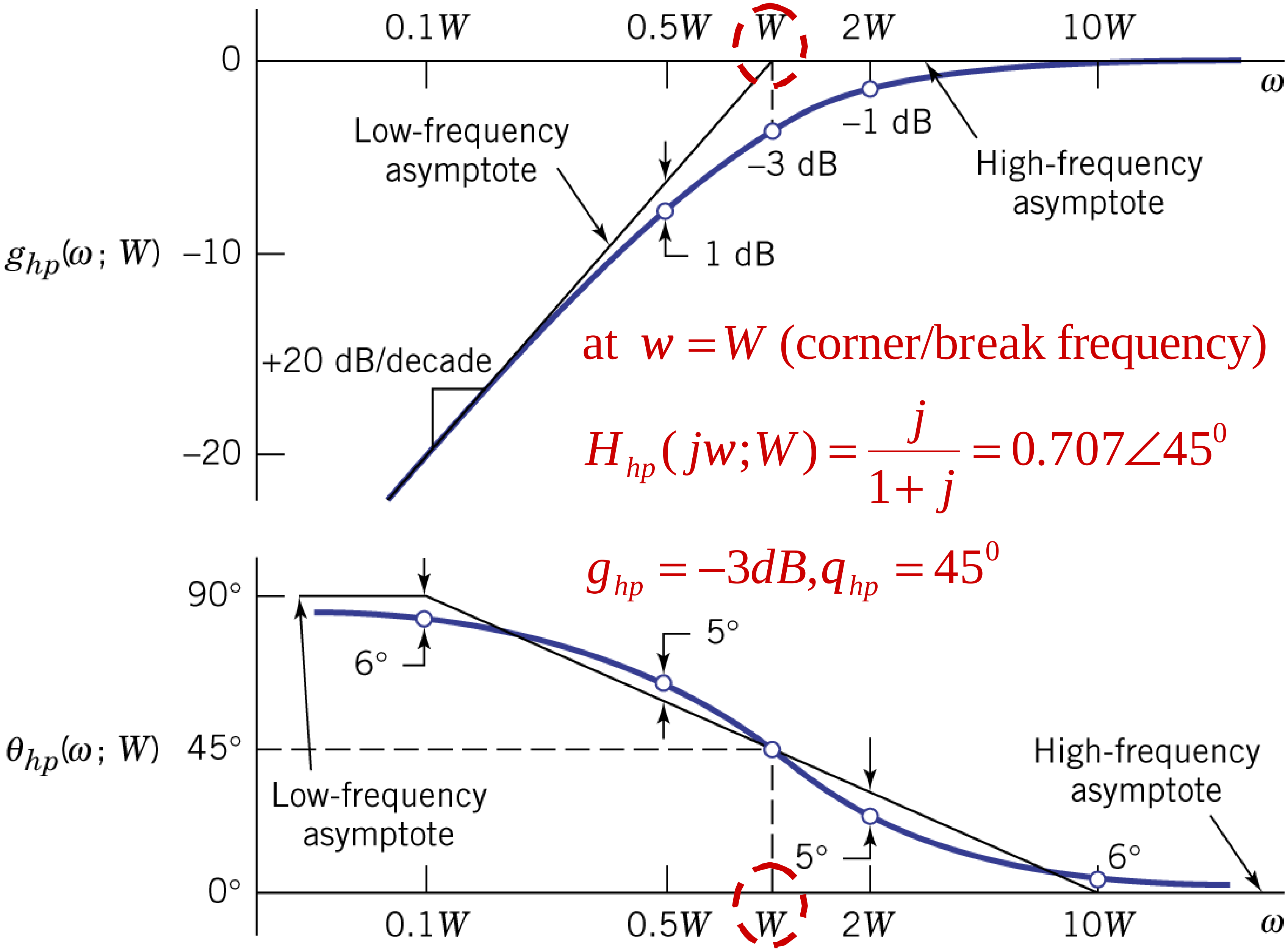
$$g_{hp} \approx 20\log\frac{w}{W}, q_{hp} = 90^0, w < 0.1W$$

$$if \quad \frac{w}{W} >> 1,$$

$$\Rightarrow H_{hp}(jw;W) \approx 1$$

$$g_{hp} \approx 0dB, q_{hp} = 0^0, w > 10W$$

Highpass Function



at $w = W$ (corner/break frequency)

$$H_{hp}(jw;W) = \frac{j}{1+j} = 0.707\angle 45^0$$

$$g_{hp} = -3dB, q_{hp} = 45^0$$

Table 11.5: Correction Terms

TABLE 11.5 Asymptote Correction Terms for H_{hp} and H_{lp}

ω/W	0.1	0.5	1	2	10
Δg (dB)	0	−1	−3	−1	0
$\Delta\theta$ (°)	−6	+5	0	−5	+6

Lowpass Function

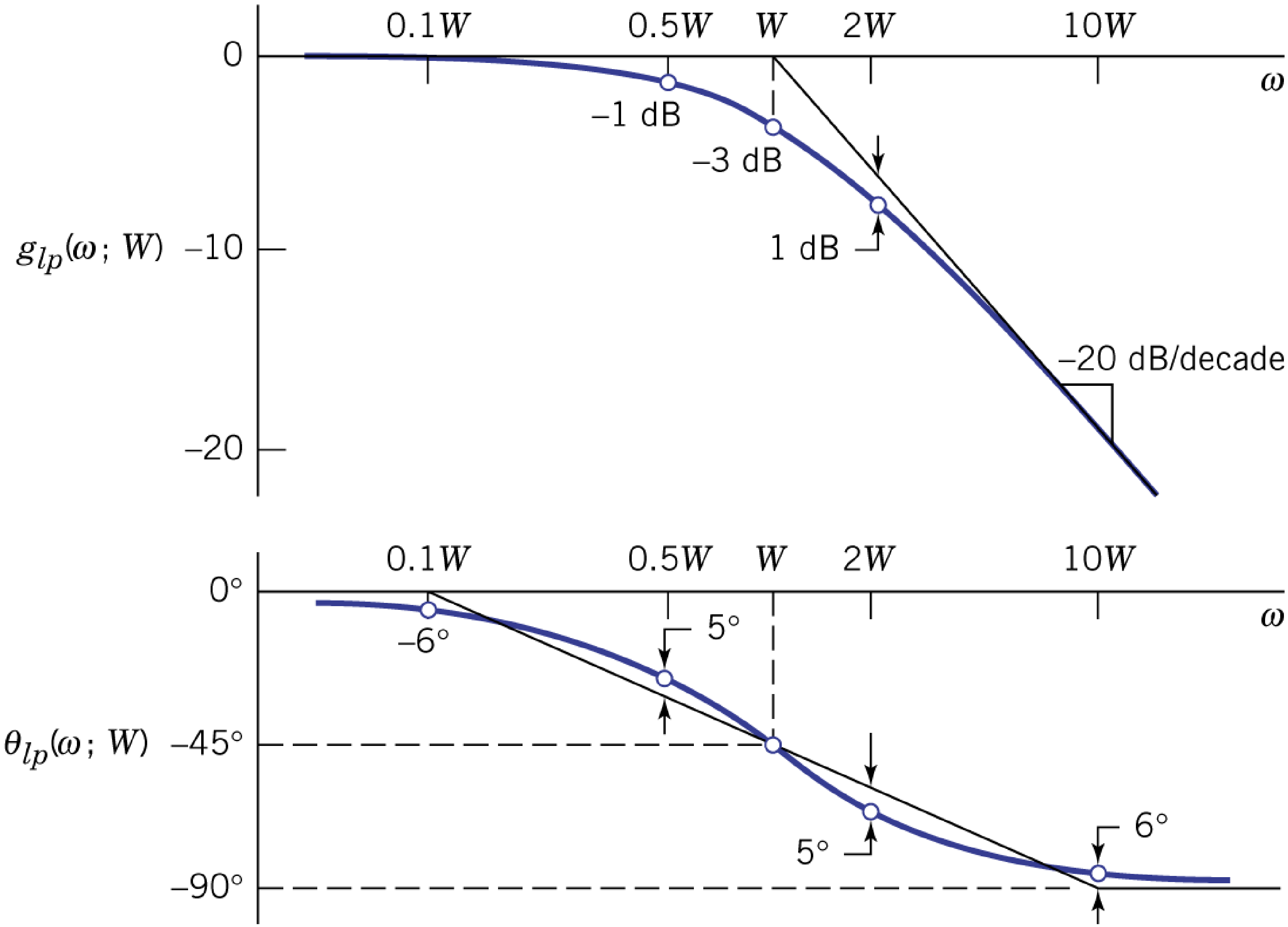
$$H_{lp}(s;W) \equiv \frac{W}{s+W} \Rightarrow H_{lp}(jw;W) \equiv \frac{1}{1+j(w/W)}$$

$$g_{lp} \approx 0dB, q_{lp} \approx 0^0, w < 0.1W$$

$$g_{lp} \approx -20\log\frac{w}{W}, q_{lp} = -90^0, w > 10W$$

$$g_{lp} \approx -3dB, q_{lp} = -45^0, w = W \text{ (break frequency)}$$

Lowpass Function



$$if \quad K \neq 1$$

$$g_{\max} = K_{dB}$$

$$g = g_{\max} - 3dB \text{ at } w = W$$

$$if \quad H(s) = H_x^m(s)$$

$$H(jw) = H_x^m(jw) = a_x^m \angle m q_x$$

$$g(w) = m \times g_x(w)$$

$$q(w) = m \times q_x(w)$$

Example 11.8: An Illustrative Bode Plot

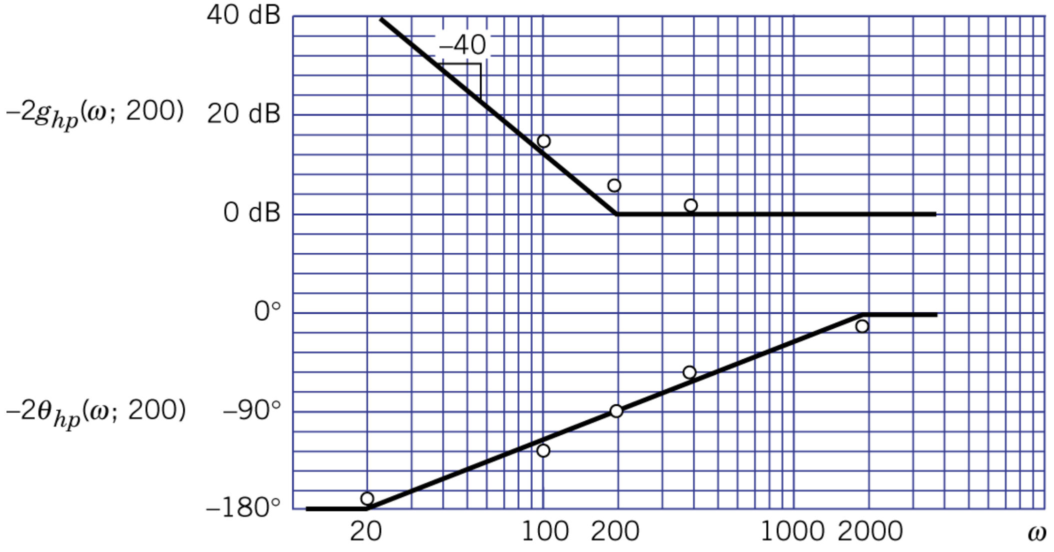
$$H(s) = -\frac{(s+200)^2}{10s^2} = -\frac{1}{10}\left(\frac{s}{s+200}\right)^{-2} = -0.1H_{hp}^{-2}(s;200)$$

$$g(w) = K_{dB} - 2g_{hp}(w;200)$$

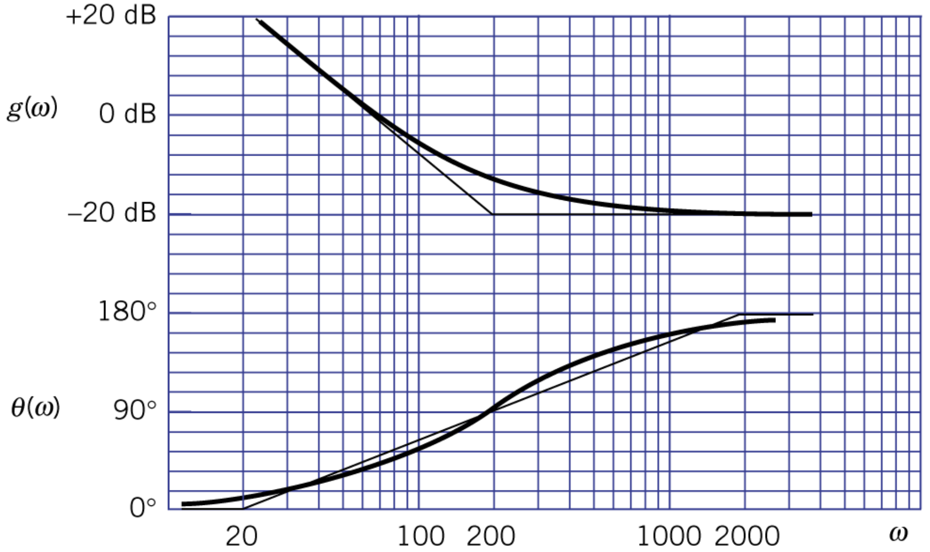
$$q(w) = \angle K - 2q_{hp}(w;200)$$

$$K_{dB} = -20dB, \angle K = \pm 180^0$$

Example 11.8: (Cont.)



(a) Asymototes and correction terms



(b) Final curves

First-Order Bode Plots

- Products of first-order factors: Bode plots of any transfer functions consisting entirely of first-order factors and powers of first-order factors can be constructed using the additive property of gain and phase. The important elements include: break frequencies, asymptotic gain and phase using straight line approximations and constants K_{dB} and $\angle K$.

Example 11.9: Frequency Response of a Bandpass Amplifier

$$H(s) = \frac{20{,}000\,s}{(s+100)(s+400)} = \frac{20{,}000}{400}\,\frac{s}{s+100}\,\frac{400}{s+400} = 50\,H_1(s)\,H_2(s)$$

$$K_{dB} = 34\,dB$$

$H_1(s)$: $H_{hp}(s;100)$ $H_2(s)$: $H_{lp}(s;400)$

	10	50	100	200	400	800	4000
Δg_1	0	-1	-3	-1	0	0	0
Δg_2	0	0	0	-1	-3	-1	0
Sum (dB)	0	-1	-3	-2	-3	-1	0

Example 11.9: (Cont.)

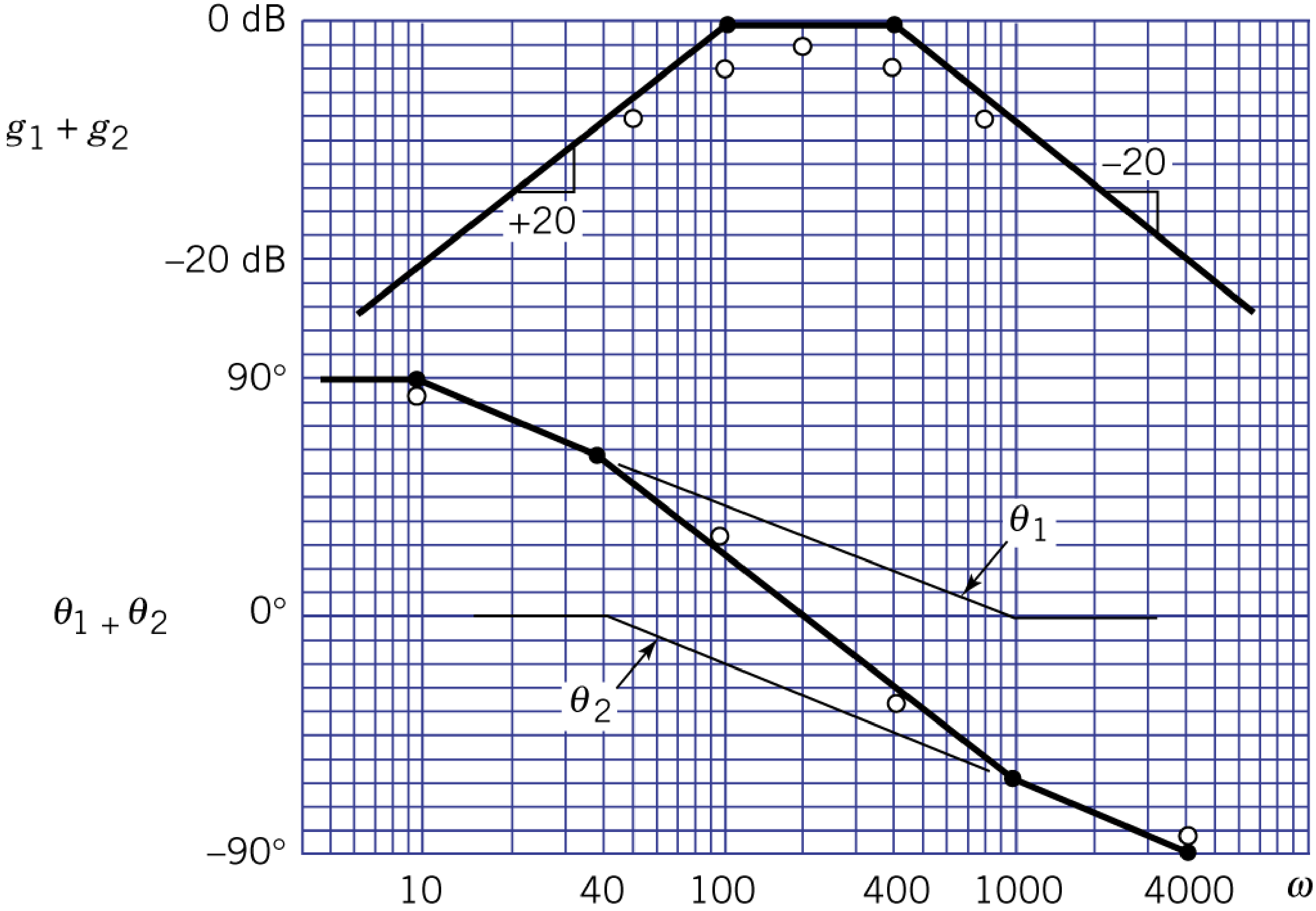
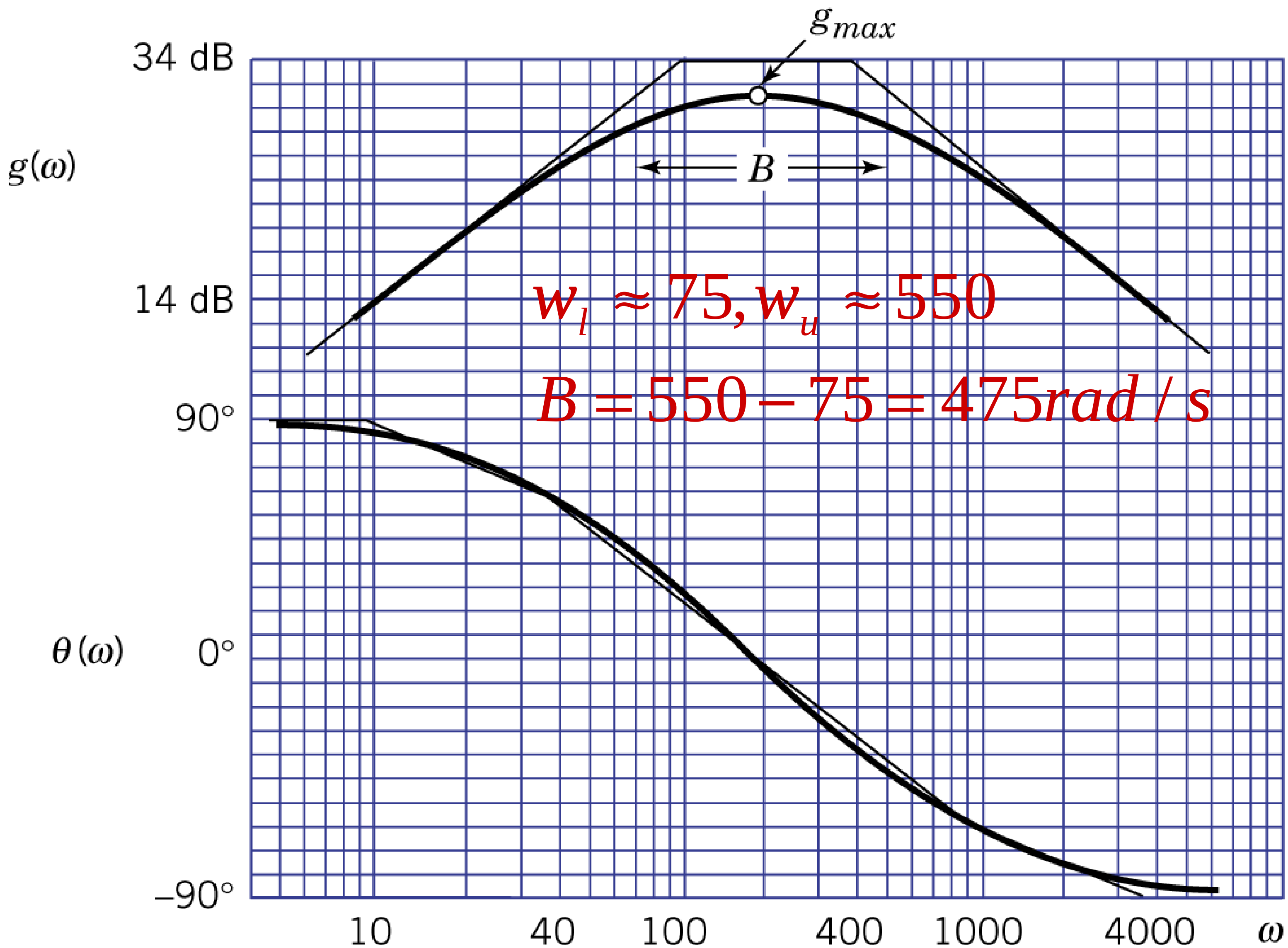


Table 11.6

TABLE 11.6

ω	10	50	100	200	400	800	4000
Δg_1	0	−1	−3	−1	0	0	0
Δg_2	0	0	0	−1	−3	−1	0
Sum (dB)	0	−1	−3	−2	−3	−1	0
$\Delta \theta_1$	−6	+5	0	−5	−4	+3*	0*
$\Delta \theta_2$	0*	−3*	+4	+5	0	−5	+6
Sum (°)	−6	+2	+4	0	−4	−2	+6

Example 11.9: (Cont.)



$w_l \approx 75, w_u \approx 550$

$B = 550 - 75 = 475 rad/s$

Second-Order Bode Plots

Quadratic Factors

- Quadratic factors for complex-conjugate poles.

$$H_q(s;w_0,Q) \equiv \frac{w_0^2}{s^2+(w_0/Q)s+w_0^2}$$

$$H_q(jw;w_0,Q) \equiv \frac{1}{1-(w/w_0)^2+j(w/Qw_0)}$$

$$H_q(jw;w_0,Q) \approx 1, \quad w << w_0$$

$$\approx -(w_0/w)^2, w >> w_0$$

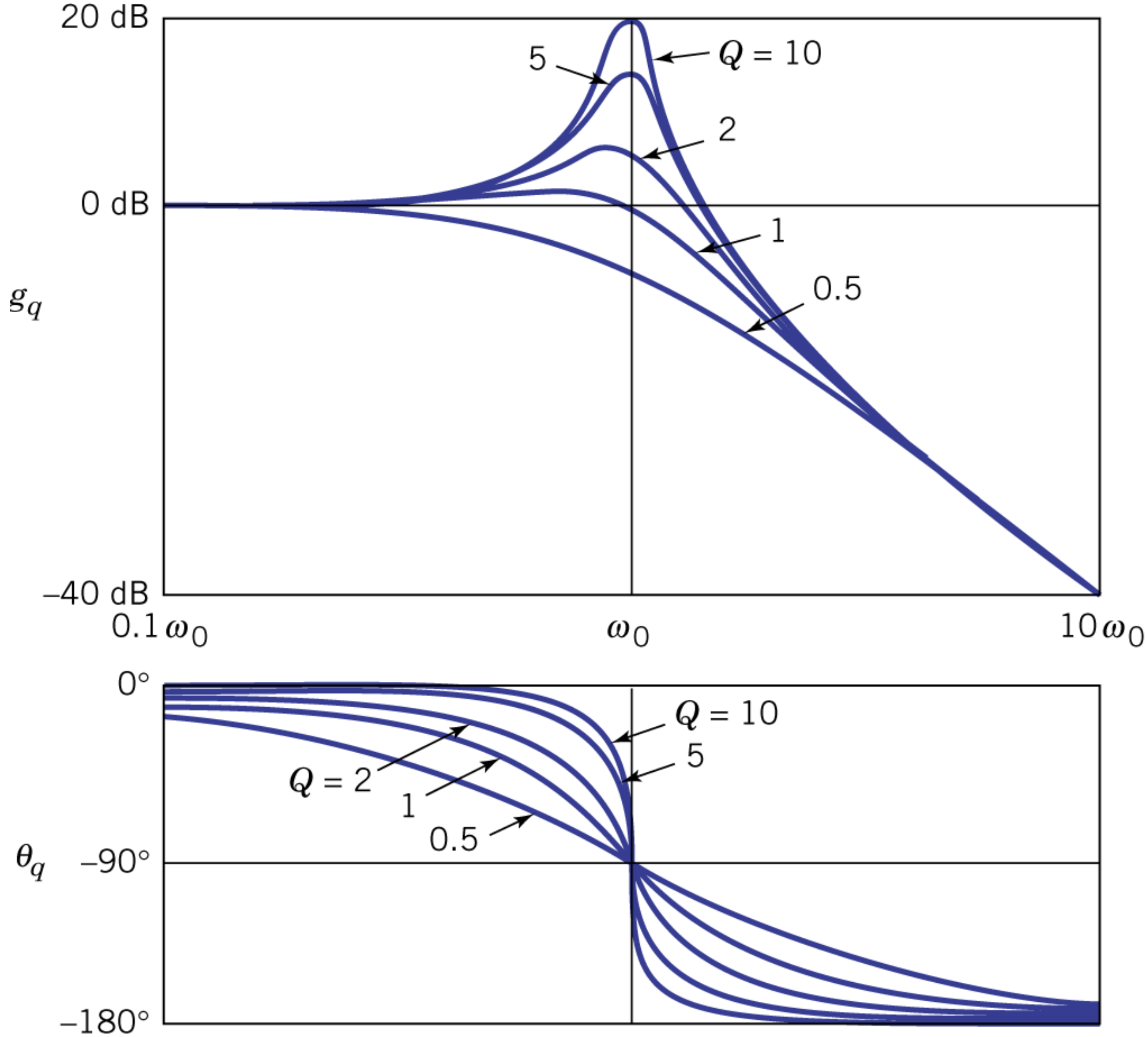
$$g_q \approx 0dB, w < 0.1w_0$$

$$\approx -40\log(w/w_0), w > 10w_0$$

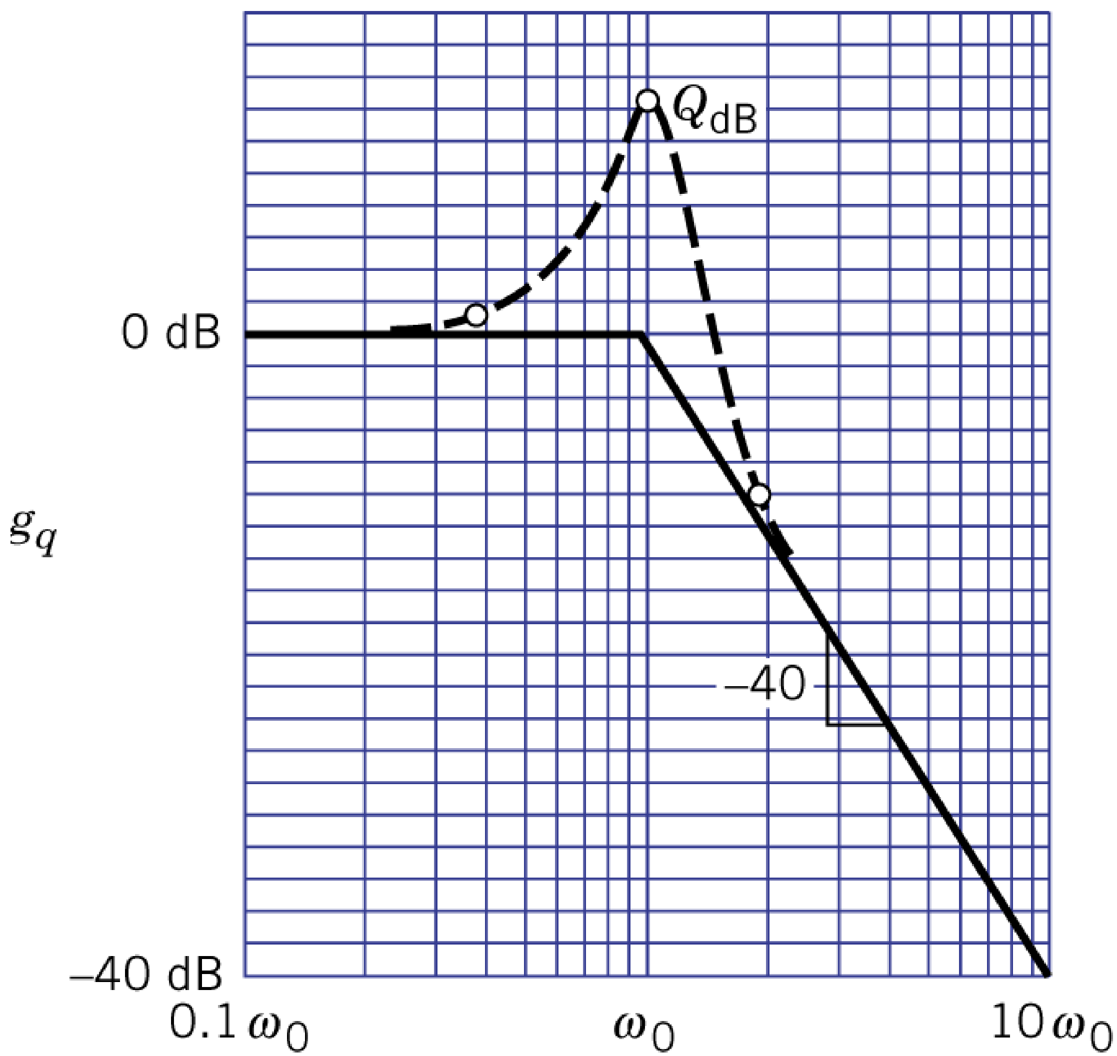
$$= 20\log Q = Q_{dB}, w = w_0$$

$$\Delta g_q = 10\log\frac{16}{9+4/Q^2}, \frac{w}{w_0} = 0.5, 2$$

Quadratic Factors



Quadratic Factors



Damping ratio :

$$\zeta = \frac{1}{2Q} = \frac{\alpha}{\omega_0}$$

$\zeta = 1$: critical damping

$\zeta < 1$: under damped

Example 11.10: Bode Plot of a Narrowband Filter

$$H(s) = \frac{20s}{s^2 + 20s + 10^4}, w_0 = 100, Q = \frac{w_0}{20} = 5, z = \frac{1}{2Q} = 0.1$$

$W = w_0 = 100$ for the ramp function

$$H(s) = \frac{20 \times 100}{10^4} \frac{s}{100} \frac{10^4}{s^2 + 20s + 10^4} = 0.2 H_r(s;100) H_q(s;100,5)$$

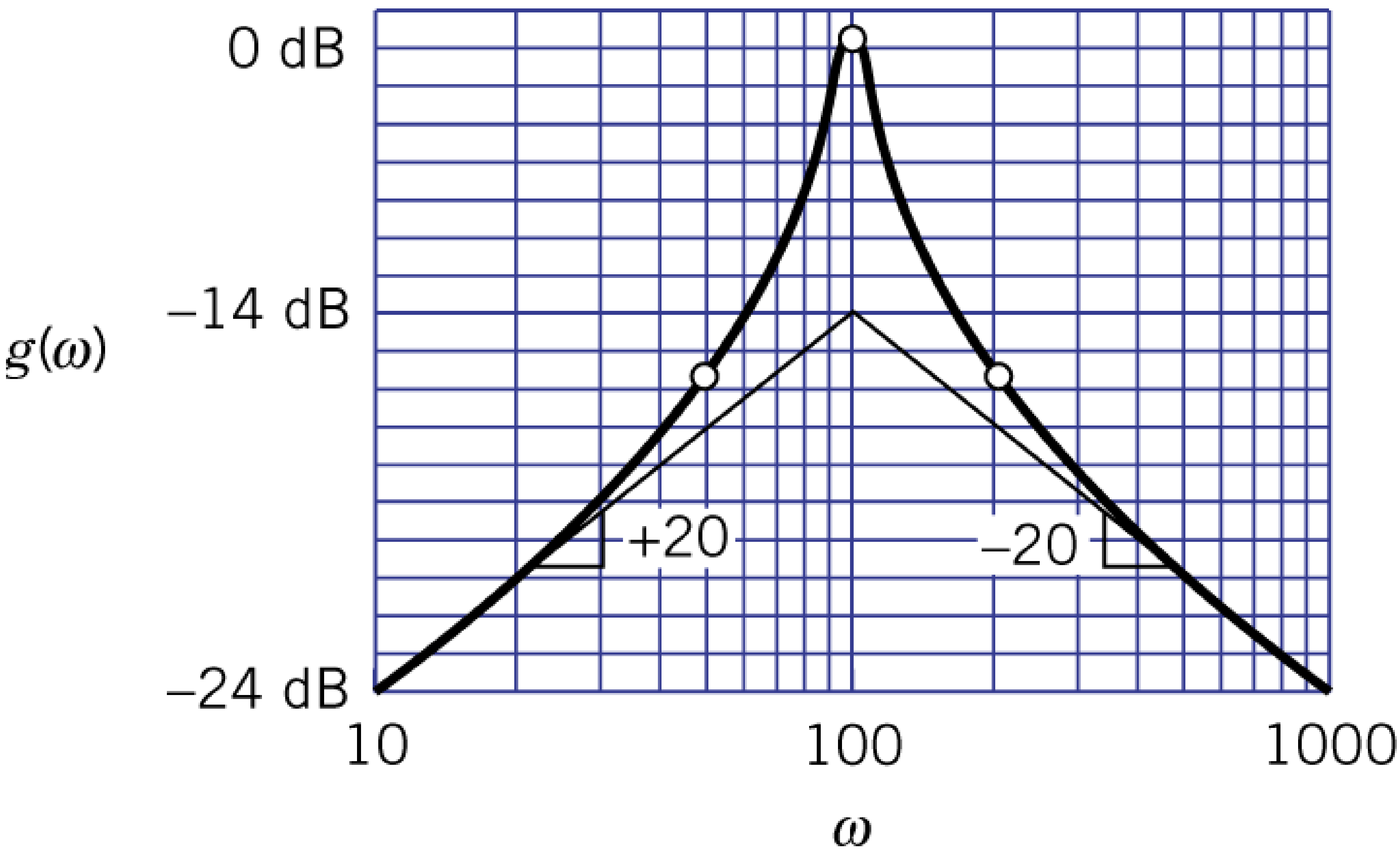
$$g(w) = -14dB + g_r(w;100) + g_q(w;100,5)$$

$$g_q = 20 \log 5 = 14, w = 100$$

$$K_{dB} = -14dB$$

$$\Delta g_q = 10 \log 1.75 = 2.4dB, w = 50,200$$

Example 11.10: (Cont.)



Chapter 11: Problem Set

- 2,6,19,22,30,35,52,59,64