CH. 13 HW

\#6.

$$\mathcal{L}[f(t/\tau)] = \int_{0^-}^{\infty} f(t/\tau)\, e^{-st}\, dt = \int_{0^-}^{\infty} f(t/\tau)\, e^{-s\tau(\frac{t}{\tau})}\, \tau\, d(\tfrac{t}{\tau})$$

$$= \tau \int_{0^-}^{\infty} f(t/\tau)\, e^{-s\tau(\frac{t}{\tau})}\, d(\tfrac{t}{\tau}) = \tau F(s\tau) \ \#$$

\#10.

$$\mathcal{L}[f(t-nT)\,u(t-nT)] = e^{-snT} F(s)$$

$$\mathcal{L}[g(t)] = F(s) + e^{-sT}F(s) + e^{-2sT}F(s) + \cdots$$

$$= F(s)\,(1 + a + a^2 + \cdots) \Rightarrow \boxed{a = e^{-sT}}$$

$$\because\ a = e^{-sT} < 1 \quad \therefore\ 1 + a + a^2 + \cdots = \frac{1}{1-a} = (1 - e^{-sT})^{-1}$$

$$\Rightarrow \mathcal{L}[g(t)] = F(s) / (1 - e^{-sT}) \ \#$$

\#16.

$$f(t) = 2t - 6u(t-2) - 2(t-3)\,u(t-3)$$

$$F(t) = 2s^{-2} - 6e^{-2s}s^{-1} - 2e^{-3s}s^{-2}$$

$$= 2(1 - 3se^{-2s} - e^{-3s})/s^2 \ \#$$

\#21.

$$sX - (-10) + X = -2Y \Rightarrow (s+1)X = -2Y - 10$$

$$2(sY - 0) + 6Y = X = (-2Y - 10)/(s+1)$$

$$\Rightarrow (2s^2 + 8s + 6 + 2)Y = -10$$

$$Y(s) = -10/2(s^2+4s+4) = -5/(s+2)^2 \Rightarrow y(t) = -5te^{-2t} \ \#$$

\#25.

$$D = s(s+4)(s+2)(s+6) \qquad F(s) = \frac{A_1}{s} + \frac{A_2}{s+2} + \frac{A_3}{s+4} + \frac{A_4}{s+6}$$

$$A_1 = \frac{-96}{2\times4\times6} = -2 \qquad A_2 = \frac{8 - 16 + 24 - 96}{-2\times2\times4} = 5$$

$$A_3 = \frac{64 - 64 + 48 - 96}{-4\times(-2)\times2} = -3 \qquad A_4 = \frac{216 - 144 + 72 - 96}{-6\times(-4)\times(-2)} = -1$$

$f(t) = -2 + 5e^{-2t} - 3e^{-4t} - e^{-6t}$ #

#28.

$F(s) = \frac{A_1}{s} + \frac{A_2}{s+4} + \frac{Bs+C}{s^2+6s+10}$

$A_1 = \frac{200}{4\times 10} = 5 \qquad A_2 = \frac{-512 + 1312 - 960 + 200}{-4(16-24+10)} = -5$

$sF(s)\Big|_{s\to\infty} = A_1 + A_2 + B = 8 \quad \Rightarrow \quad B = 8$

$F(1) = A_1 + \frac{A_2}{5} + \frac{B+C}{17} = \frac{530}{5\times 17}$

$\Rightarrow C = 30$

$\alpha = \frac{6}{2} = 3, \quad \beta^2 = 10 - 3^2 = 1 \Rightarrow \beta = 1$

$K = B + j\frac{\alpha B - C}{\beta} \qquad (13.27)$

$= 8 + j\frac{3\times 8 - 30}{1} = 8 - j6 = 10\angle{-37^\circ}$

$f(t) = 5 - 5e^{-4t} + 10e^{-3t}\cos(t - 37^\circ)$ #

#31.

$F(s) = \frac{A_{10}}{(s+3)^2} + \frac{A_{11}}{s+3} + \frac{A_3}{s+4} \qquad A_3 = \frac{64 - 48 - 10}{1} = 6$

$sF(s)\Big|_{s\to\infty} = 4 = A_{11} + A_3 \quad \Rightarrow \quad A_{11} = -2$

$F(0) = \frac{-10}{9\times 4} = \frac{A_{10}}{3^2} + \frac{A_{11}}{3} + \frac{A_3}{4} \Rightarrow A_{10} = -10$

$f(t) = -10te^{-3t} - 2e^{-3t} + 6e^{-4t}$ #

#35.

$s^2 + 2s + 5 = (s+1-j2)(s+1+j2)$

$D = [(s+1-j2)(s+1+j2)]^2$

$F(s) = \frac{A_{10}}{(s+1-j2)^2} + \frac{A_{11}}{s+1-j2} + \frac{A_{10}^*}{(s+1+j2)^2} + \frac{A_{11}^*}{s+1+j2}$

$A_{10} = \frac{-2s^2 - 2s + 6}{(s+1+j2)^2}\Bigg|_{-1+j2} = -1 \qquad A_{10}^* = -1$

$$A_{11} = \left. \frac{(s+1+j2)(-4s-4) - 2(-2s^2-4s+6)}{(s+1+j2)^3} \right|_{-1+j2} = 0, \quad A_{11}^* = 0$$

$$f(t) = -t e^{(-1+j2)t} - t^{(-1-j2)t} = 2t\, e^{-t} \cos(2t + \pi) \quad \#$$

\# 38.

$$F(s) = \frac{A_{i0}}{(s+s_i)^4} + \frac{A_{i1}}{(s-s_i)^3} + \frac{A_{i2}}{(s-s_i)^2} + \frac{A_{i3}}{s+s_i} + F_1(s)$$

$$(s-s_i)^4 F(s) = A_{i0} + (s-s_i) A_{i1} + (s-s_i)^2 A_{i2} + (s-s_i)^3 A_{i3} + (s-s_i)^4 F_1(s)$$

$$d[(s-s_i)^4 F(s)]/ds = A_{i1} + 2(s-s_i) A_{i2} + 3(s-s_i)^2 A_{i3} + 4(s-s_i)^3 F_1(s)$$

$$d^2[(s-s_i)^4 F(s)]/ds^2 = 2A_{i2} + 6(s-s_i) A_{i3} + 12(s-s_i)^2 F_1(s)$$

$$d^3[(s-s_i)^4 F(s)]/ds^3 = 6A_{i3} + 24(s-s_i) F_1(s)$$

$$\Rightarrow A_{i0} = [(s-s_i)^4 F(s)]\Big|_{s=s_i} \qquad A_{i1} = \frac{d}{ds}[(s-s_i)^4 F(s)]\Big|_{s=s_i}$$

$$A_{i2} = \frac{1}{2!}\frac{d^2}{ds^2}[(s-s_i)^4 F(s)]\Big|_{s=s_i}$$

$$A_{i3} = \frac{1}{3!}\frac{d^3}{ds^3}[(s-s_i)^4 F(s)]\Big|_{s=s_i} \quad \#$$

\# 42.

$$\mathcal{L}[f''(t)] = s^2 F(s) - sf(0^-) - f'(0^-)$$

$$f''(0^+) = \lim_{s\to\infty} s \frac{s^2 N(s) - [s f(0^-) + f'(0^-)] D(s)}{D(s)} \quad \#$$

\# 46. Pole at $s=1$, so $f(\infty)$ does not exist.

$$sF(s) = (2s^3 + 9s^2 + 16) / (s^3 + 7s^2 + 8s - 16)$$

$$\Rightarrow f(0^+) = 2 \quad \#$$

$$[s^2 N(s) - 2s D(s)]/D(s) = \frac{(2s^5 + 9s^4 + 16s^2 - 2(s^5 + 7s^4 + \cdots))}{(s^4 + \cdots)}$$

$$= \frac{[(9-14)s^4 + \cdots]}{(s^4 + \cdots)} \Rightarrow f'(0^+) = -5 \quad \#$$

#51

$$X(s) = \frac{1}{s} - \frac{1}{s+3} = \frac{3}{s(s+3)}$$

$$Y(s) = \frac{6s+90}{(s^2+8s+15)\,s(s+3)} = \frac{A_1}{s} + \frac{A_{20}}{(s+3)^2} + \frac{A_{21}}{s+3} + \frac{A_4}{s+5}$$

$$A_1 = \frac{90}{3^2 \times 5} = 2 \qquad A_{20} = \frac{-18+90}{-3\times 2} = -12$$

$$A_{21} = \left.\frac{(s^2+5s)6 - (6s+90)(2s+5)}{(s^2+5s)^2}\right|_{-3} = 1 \qquad A_4 = \frac{-30+90}{-5(-2)^2} = -3$$

$$y(t) = 2 - 12te^{-3t} + e^{-3t} - 3e^{-5t} \; *$$

#57.

$$X(s) = 5\,\frac{0\times s - 3\times 1}{s^2+3^2} = \frac{-15}{s^2+3^2}$$

$$Y(s) = \frac{-15\,\cancel{(s^2+9)}}{s(s^2+2s+5)\,\cancel{(s^2+3^2)}} = \frac{-15}{s(s^2+2s+5)}$$

$$= \frac{A_1}{s} + \frac{Bs+C}{s^2+2s+5}$$

$$A_1 = -15/5 = -3, \quad sY(s)\big|_{s\to\infty} = 0 = A_1 + B \Rightarrow B = 3$$

$$Y(1) = -15/8 = A_1 + (B+C)/8 \Rightarrow C = 6$$

$$\alpha = 1, \quad \beta^2 = 5 - 1^2 = 2^2 \Rightarrow \beta = 2$$

$$K = 3 + j\frac{(3-6)}{2} = 3 - j1.5 = 3.35\angle{-26.6^\circ}$$

$$H(j3) = 0 \Rightarrow y_F(t) = 0,$$

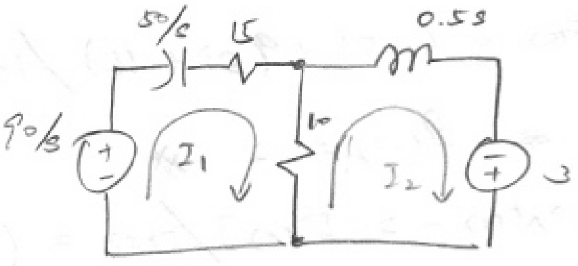
$$y(t) = -3 + 3.35e^{-t}\cos(2t - 26.6^\circ) \; *$$

#58

$$i_L(0^-) = 6$$

$$Li_L(0^-) = 3$$

$$V_C(0^-) = 15\times 6 = 90$$



$$\begin{bmatrix} 25 + \frac{50}{s} & -10 \\ -10 & 0.5s+10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 90/s \\ 3 \end{bmatrix}$$

$$I_1(s) = \frac{6s+72}{s^2+14s+40} = \frac{A_1}{s+4} + \frac{A_2}{s+10}$$

$A_1 = 8$, $A_2 = -2$, $i_1(t) = 8e^{-4t} - 2e^{-10t}$ #

#63.

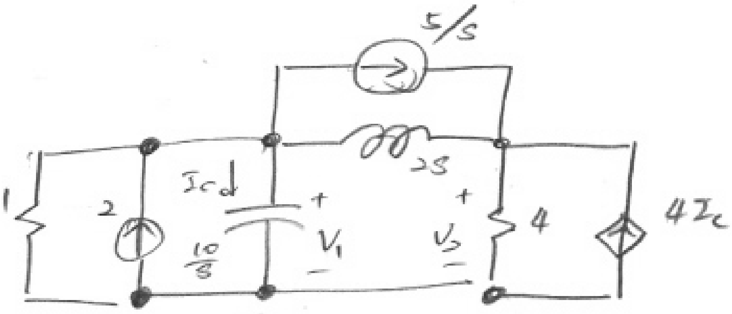
$i_o(0^-) = 0$

$i_L(0^-) = 25/(1+4) = 5$

$V_c(0^-) = 4\, i_L(0^-)$

$C v_c(0^-) = 2$

$I_c = sV_1/10$



$$\begin{bmatrix} 1/1 + s/10 + 1/2s & -1/2s \\ -1/2s & 1/4 + 1/2s \end{bmatrix} = \begin{bmatrix} 2 - 5/s \\ 5/s \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 4s/10 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$V_1(s) = (20s - 10)/(s^2 - 6s + 25) \Rightarrow B = 20, \; C = -10$

$\alpha = -6/2 = -3$, $\beta^2 = 25 - 3^2 = 4^2 \Rightarrow \beta = 4$

$$K = 20 + j\,\frac{-3\times 20 - (-10)}{4} = 20 - j\,12.5 = 23.6\angle{-32^\circ}$$

$V_2(t) = 23.6\, e^{3t} \cos(4t - 32^\circ)$ #

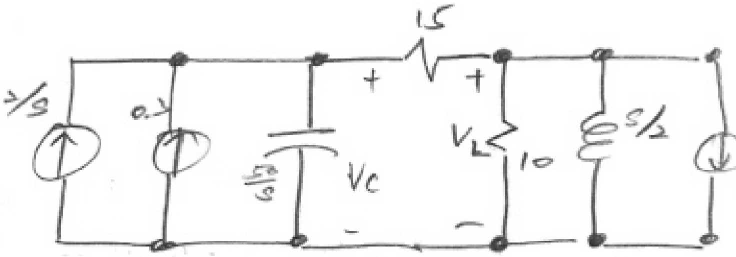
#65.

$i_L(0^-) = 1$

$V_c(0^-) = 15 \times 1 = 15$

$C v_c(0^-) = 0.3$

$I_s(s) = \frac{2}{s}$



$$\begin{bmatrix} s/50 + 1/15 & -1/15 \\ -1/15 & \frac{2}{s} + \frac{1}{10} + \frac{1}{15} \end{bmatrix} \begin{bmatrix} V_c \\ V_L \end{bmatrix} = \begin{bmatrix} 0.3 + 2/s \\ -\frac{1}{s} \end{bmatrix}$$

$V_L(s) = 20/(s^2+14s+40)$, $sV_L(s) = 20s/(s^2 + \dots) \Rightarrow V_L(0^+) = 0$ #

$V_L(0^-) = 0$, $s^2N/D = 20s^2/(s^2 + \dots) \Rightarrow V_L'(0^+) = 20$ #